Middle Level Math:

Strategies to Improve Teaching Mathematics Vocabulary

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STRATEGIES TO IMPROVE TEACHING MATHEMATICS VOCABULARY

By

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Abstract

This extensive literature review explores the research on teaching mathematical vocabulary in the middle level classroom. The first goal was to identify specific research about the most effective strategies for teaching mathematical vocabulary. The second goal was to find strategies to teach multiple-meaning mathematical words. The final goal was to uncover research-based strategies that help students make connections between mathematics vocabulary and their lives. The research indicates that reading and mathematics vocabulary needs to be taught differently due to the specificity of mathematical vocabulary. This paper presents a plethora of strategies for teaching mathematical vocabulary including ideas for teaching multiple-meaning words and ways of helping students make connections between mathematical vocabulary and their lives. This paper also includes some recommendations and resources for teachers looking to improve their practice.

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Introduction

"I thought the word quotient meant any problem, when it actually means a division problem. So a negative divided by a negative would be a positive."

"I didn't know the definition of congruent so I couldn't come up with an answer."
"I forgot what congruent meant so I guessed."

"I thought the word sum meant multiply so I did -3 x 4 which always comes out negative."
(Sprosty, 2011) (Sprosty, 2011.)

Knowing the correct vocabulary in mathematics is essential. Not knowing the vocabulary terms can be a hindrance for a student solving any mathematical problem. The above quotes were taken from Kelli Sprosty's seventh grade students who were trying to explain their mistakes on the *Ohio Mathematics Achievement* practice test. These comments demonstrate the problems that can occur when students do not have a firm knowledge or understanding of mathematical vocabulary (Sprosty, 2011) (Sprosty, 2011.) . Without an adequate knowledge of the vocabulary, students may struggle with answering mathematical questions, even if they understand the associated concepts and procedures.

If students cannot understand what the question is asking of them, or in essence, the vocabulary, they may not be successful in answering the question. The language of mathematics is a critical factor in students' understanding the subject (Aiken, 1972; Hersh, 1997; Ogilvy, 1949; Rubenstein, 2007). What research is available concerning teaching mathematical vocabulary? What are the most effective ways to teach mathematics vocabulary? What are effective ways to teach students about multiple-meaning words and how would a teacher best teach multiple-meaning words during a mathematics class? How might a teacher help students make mathematical vocabulary connections to their own lives? These are the 'drive–a-teacher-crazy' questions that I had that led me to this research topic.

Problem Statement

The problem being researched in this paper is finding effective strategies to help students learn the mathematical vocabulary that is necessary for their success. Word recognition can be used as a predictive measure for reading comprehension in later grades; therefore, vocabulary knowledge in the primary grades is important for reading success in later grades (Scarborough, 1998). In addition, the correlation between vocabulary knowledge and verbal ability is strong (Sternberg, 1987). This correlation between vocabulary knowledge and verbal ability in reading can be extended to mathematical vocabulary knowledge and verbal ability in understanding mathematics concepts related to vocabulary. If a student has reading and language processing difficulties, it certainly causes problems in mathematics classes (Montis, 2000) .

Teachers know that teaching vocabulary is tremendously important, but forty years ago, teachers did not know much about effective ways to teach vocabulary (Graves, 2009). Today, there are many researchers studying the best ways to teach vocabulary. There is ample information and data that supports the importance of teaching vocabulary. Although we certainly do not know everything there is to know about vocabulary and teaching vocabulary, we know a great deal (Barwell, Leung, Morgan, & Street, 2002; Graves, 2009). As a teacher, I see the problem that not knowing the mathematical vocabulary causes in my classroom and recognize the importance of teaching mathematical vocabulary. This has led me to study this issue.

Purpose

The purpose of this study is to research and report findings pertaining to mathematics vocabulary in the form of an extensive literature review. The research literature suggests that there is a problem with teaching and learning mathematical vocabulary. It is widely accepted that vocabulary knowledge is essential for reading (Biemiller, 2001); however, it is just as essential for learning mathematics. Chard (n.d.), a language and vocabulary consultant for Houghton Mifflin and math professor of reading at the University of Oregon, says that although vocabulary is not the only condition needed for success in mathematics, it is a very necessary component. Vocabulary is a vital link between a child's sense of number and order and her understanding of mathematical concepts. Vocabulary in reading classes is supported by pictures or contextual sentence clues. In mathematics instruction, new vocabulary is often supported by some kind of table or graphic to help with understanding. However, when it comes to some abstract mathematical concepts, there are often no visual graphics that can be used to explain the mathematical concepts (Barwell, Leung, Morgan, & Street, 2002; Chard, n.d.). Granted, children may grasp the concepts of quantity and other relational concepts from a very early age simply through everyday exposure (Dehaene, 1997) , but there are many mathematical concepts that must be taught and students likely will not learn without meaningful practice.

Teaching the language of mathematics is an important part of mathematics instruction. Mathematics vocabulary plays several important roles in our classrooms. Foremost, we teach using spoken and written language. It is our major means of communication. Students also need to be able to communicate about mathematics. This means more than just verbalizing word problems. They need to understand mathematical information that is read, written, or presented orally. An essential aspect of this communication is the understanding and use of mathematical language as students build understanding as they process mathematical ideas through language (Pugalee, 2001; Zaslavsky & Shir, 2005). Pugalee (2001) studied the incorporation of mathematical communication into middle school classrooms and found that, "When students are given the opportunity to communicate about mathematics, they engage thinking stills and processes that are crucial in developing mathematical literacy" (Pugalee, 2001, p. 296).

Even though written and oral mathematical language is the way that we communicate in mathematics classes, both are often a challenge for our students. Teachers sometimes forget that the words and phrases used while teaching may seem foreign to students. Kotsopoulos' (2007), research with a ninth grade class found that there are vast differences between what a teacher believes students know and what they really know, and that it is critical for students to master this language. This mastery of vocabulary is important for students to be able to read, understand, and discuss mathematical ideas. Unlike everyday English, the language of mathematics is usually limited to school. Even more specifically, it is often used only during mathematics class. Because of this, teachers need to be aware of many issues related to mathematical language and students' growing fluency with it (Kotsopoulos, 2007; Thompson & Rubenstein, 2000)

Research Questions

The questions that are guiding this extensive literature review are:

1. What are effective strategies for teaching mathematics vocabulary?
2. How might a teacher best teach vocabulary with multiple meanings during mathematics instruction?
3. How can a teacher best help students make connections between mathematical vocabulary and their lives?

Teaching and learning mathematical vocabulary is difficult, but having a working knowledge of the vocabulary can help students understand, express, and explain their thinking and knowledge of mathematical concepts. The field of vocabulary research is fairly new, but it is also quite large. Because of this, I have decided to limit research for this project to research specifically about strategies for teaching and learning mathematical vocabulary. There are many views about teaching mathematics vocabulary, but in many classrooms we are just expecting children to ‘just pick it up as we go.’ This does not seem to be working in my classroom. I wanted to find effective strategies for teaching mathematical vocabulary that I can use regularly to help my students be more comfortable in using vocabulary a part of their oral and written explanations.

Chapter 2

Literature Review

This section of the paper provides researched information about mathematical vocabulary. It will explore effective strategies for teaching mathematics vocabulary, vocabulary with multiple meanings, and connections between mathematical terms and students' existing vocabulary and knowledge.

The Power of Words

Students need to become aware of the words they read, hear, speak, and use in writing. There is power in words. Having an appreciation of that power can help students understand why certain words are used in different situations (Carter & Dean, 2006; Graves 2009).

> It might be that problems of vocabulary are considered to be fairly superficial within the whole issue of language and mathematics learning, but it is nevertheless critical that such problems are not ignored in the hope they will go away. (Orton, 1987, p. 127)

Do your students speak mathematics? Do they think that the mathematics classroom is a foreign country where they must use a foreign language (Kotsopoulos, 2007) ? Are they sometimes confused or overwhelmed by new vocabulary? Do they misuse words, forget key terms, or ignore important distinctions between words? Thompson and Rubenstein (2000), teachers of pre-service and master's degree students at the University of Michigan, found that when they were focusing on mathematics communication and curriculum, students asked questions such as, "Where did anyone ever get a strange word like *asymptote*?" or "I forget, is twelve a factor or a multiple of twenty-four?" (p. 568). Reuben Hersh, a University of New

Mexico professor who studies mathematics as a part of human culture, coined the words *math lingo* to explain that mathematics has a language all its own (Hersh, 1997).

Reading mathematics textbooks can be difficult when the concepts are new. In addition, when students do not have access to the meanings of the mathematical vocabulary and specialized terms and symbols, they face the challenging, if not impossible, task of reading and comprehending mathematics texts (Roe, 1995). Precise meanings of vocabulary must be very clear to readers in order for comprehension to occur. If comprehension does not occur, the entire meaning of the passage will most likely be altered (Reehm & Long, 1996). Unlike other types of written material that students encounter, mathematical texts do not have many contextual clues to help decode the meaning of most specialized words (Reehm & Long, 1996).

As students move into the middle grades and take their first algebra or pre-algebra course, the language of mathematics may become even more confusing. In these courses, students encouter many new concepts and vocabulary. The vocabulary words may sound familiar, but the mathematical definitions are not the same as the vocabulary definitions the students know. For example, students may know words like power, base, and expression, but in mathematics, these words take on new meanings (Barwell, Leung, Morgan, & Street, 2002; Carter & Dean, 2006; Gay, 2002).

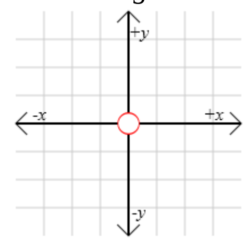
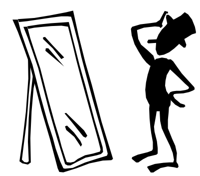
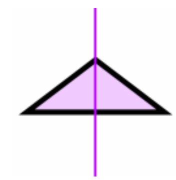
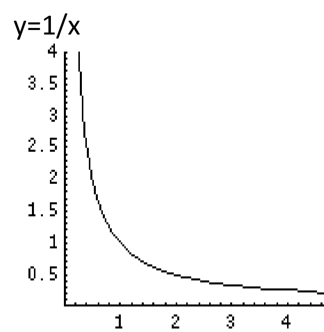
Analysis of Mathematical Language

Kane, Byrne, and Hater (1974), Kane (1974), Shuard and Rothery (1984), and Carter (2006) highlight many vocabulary issues to consider when comparing mathematical language to everyday English. Table 1 summarizes these issues and includes others identified by Thompson and Rubenstein (2000). Language issues identified in the table do not occur in isolation. Many, if not all, can occur in a single class period. Table 1 shows potential vocabulary problems caused by language difficulties that are often overlooked as we teach mathematics vocabulary.

The first illustrative example shows how the word *origin* in everyday language means where something begins. In mathematical contexts, *origin* has a very specific meaning. The *origin* of a graph is the point specified by the ordered pair (0, 0). It is where both the x-coordinate and the y-coordinate are zero and their respective axes intercept. The second example shows an everyday definition of a reflection in a mirror, while reflection symmetry (sometimes called *line symmetry* or *mirror symmetry*) is easy to recognize, because one half is the reflection of the other half, very similar to the first picture. The third illustrative example and graph shows that the word asymptote only appears in a mathematical context. The fourth illustrative example shows two definitions within mathematics that are both definitions of the word *square*. The fifth illustrative example compares an equation and an expression. It shows that an equation has an equal sign, which can be easily confused with an expression, which does not have an equal sign. Equations can be solved, finding an answer for x, but an expression can only be simplified.

Table 1

Vocabulary Issues and Examples

Category of Potential Pitfalls	Examples	Illustrative Examples	
Some words are shared by mathematics and everyday English, but they have distinct meanings.	*Number*: prime, power, factor *Algebra:* origin, function, domain, radical, imaginary *Geometry*: volume, leg, right *Statistics/probability*: mode, event, combinations *Discrete mathematics*: tree	*Everyday use* *Origin* the point at which something begins or rises or from which it derives	*Mathematical use* *Origin* 
Some mathematics words are shared with English and have compatible meanings, but the mathematical meaning is more precise.	*Number*: divide, equivalent, even difference *Algebra*: continuous, limit, amplitude, slope *Geometry*: similar, reflection *Statistics/probability*: average *Discrete mathematics*: array, edge	*Everyday use* *Reflection* 	*Mathematical use* *Reflection* 
Some mathematical terms are found only in a mathematical context.	*Number*: quotient, decimal, denominator, algorithm *Algebra:* asymptote, integer, hyperbola *Geometry*: quadrilateral, parallelogram, isosceles, hypotenuse *Statistics/probability*: outlier, permutation *Discrete mathematics*: contra-positive		*Mathematical use* *Asymptote*-An asymptote is, essentially, a line that a graph approaches, but does not intersect. For example, in the following graph of y=1/x, the line approaches the x-axis (y=0), but never touches it. No matter how far we go into infinity, the line will not actually reach y=0, but will always get closer and closer.

:

Some words have more than one mathematical meaning.	*Number*: inverse, round *Algebra:* square, range, base, inverse, degree *Geometry*: square, round, dimensions, median, base, degree, vertex *Statistics/probability*: median, range *Discrete mathematics*: dimensions, inverse, vertex	*Definition 1* *Square* *A 4-sided regular polygon with all sides equal and all internal angles 90°* 	*Definition 2* *Square* $5^2=25$
Some mathematical terms sound like everyday English words.	*Number*: sum or some *Algebra:* sine or sign, cosine or cosign *Geometry*: pi or pie, dual or duel, plane or plain *Statistics/probability*: leaf, as in stem-and-leaf or leave *Discrete mathematics*: complement or compliment, graph or graft		
Some mathematical words are related, but students confuse their distinct meanings.	*Number*: factor and multiple, hundreds and hundredths, numerator and denominator *Algebra:* equation and expression, solve and simplify *Geometry*: theorem and theory *Statistics/probability*: dependent events and independent events *Discrete mathematics*: converse, inverse, and contrapositive	*Equation* $5 + 3 = x$ $x = 8$ *solve an equation*	*Expression* $5 + 3$ 8 *simplify an expression*

Note. Adapted from "Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies", Thompson and Rubenstein, 2000. Used with permission.

Teaching and learning mathematical vocabulary is difficult; however, it is essential. Having a working knowledge of the vocabulary can help students understand, express, and explain their thinking and knowledge of mathematical concepts.

Strategies for Teaching Mathematical Vocabulary

Vocabulary instruction is a well-researched field (Graves, 2009; Thompson & Rubenstein, 2000) and instructional strategies have been designed that enhance vocabulary learning. This next section will begin with an explanation of aspects common to nearly all strategies followed by explanations of general pedagogical strategies for teaching vocabulary and then explanations of more specific strategies.

Common aspects in teaching vocabulary. Teaching vocabulary involves time and commitment. Kucan, Trathen, Straits, Hash, Link, and Miller (2007), a group of university professors, formed a collaborative effort with secondary teachers at Allegheny High School to study the effects of vocabulary instruction. Four-hundred twelve high school students participated in this study. During the study, entitled, A *Professional Development Initiative for Developing Approaches to Vocabulary Instruction with Secondary Mathematics, Art, Science, and English Teachers*, the teachers came up with ten common aspects they identified as criteria for successfully teaching vocabulary to students:

1. Teacher commitment to vocabulary development in terms of planning and class time;
2. Willingness to experiment with a variety of instructional approaches and to adapt those approaches as needed;
3. Setting learning goals in terms of developing rich representations of word meanings as well as an understanding of how words work;
4. Facilitating student access to multiple sources of information;
5. Providing support and encouragement for students to discover connections among words, including forms of words and related words;

6. Giving students opportunities to create multiple representations of words;
7. Highlighting cross-curricular connections;
8. Sustaining commitment to activity-based approaches;
9. Acknowledging the social dimension of classrooms by providing chances for students to work together and to present and perform with and for their peers;
10. Developing interesting assessments involving multiple contexts for focusing on word meanings and features of words. (Kucan, et al., 2007, p. 10) .

General Strategies for Teaching Mathematical Vocabulary

Building concepts first. Thompson and Rubenstein (2000) suggest that building concepts first, then attaching vocabulary to establish the idea is one of the simplest, yet most effective ways to teach mathematical vocabulary. For example, a teacher could ask geometry students to sort several quadrilaterals and then identify the categories such as those with exactly two pairs of parallel sides. After students have identified these shapes, the name *parallelogram* can then be attached to that category. After introducing new vocabulary, another very simple step should be taken: to have students say, write, and spell the word clearly. Students can then record the new word and its meaning with a drawing in their personal mathematics journal (Thompson & Rubenstein, 2000).

Manouchehri (2007) suggests that math instruction should provide opportunities for students to make meaning through classroom discussion. During the second week of school, she gave her secondary mathematics students several problems. They worked on the problems in small groups and then presented their solutions to the class. During the presentations, the students were directed to ask questions and make comments about the strategies of each group's presentation of their problem solving strategies. The focus of the exercise was not so much on teaching specific vocabulary, but rather on the language of mathematics that her students used to explain and defend their ideas. She concluded that the experience helped students to construct meaning and make connections to their own lives and to enrich their learning (Manouchehri, 2007). Although this particular example was in a high school setting, this type of *building the concept first* could be done with upper elementary or middle school students.

Oral strategies. Students must have more than just one opportunity to say, write, and spell new mathematics vocabulary. "Students must encounter words in context more than once to learn them" (Marzano, Pinkering, & Pollock, 2001, p. 124) (Marzano, Pinkering, & Pollock, 2001, p. 124). They need to *own* the language and use it comfortably. This language fluency requires much language usage. Thompson and Rubenstein (2000) suggest that discussing concepts using the new vocabulary while problem-solving offers the opportunity for students to *math talk*. Teachers should listen to the *math talk*, reinforce correct use of vocabulary, and help students correct or rephrase mathematical ideas as needed. Small groups offer easier practice than trying new vocabulary in front of the whole class (Thompson & Rubenstein, 2000).

Writing strategies. There is also value in having students write about mathematics (Burton & Morgan, 2000). Writing involves a higher level of communication than just oral communication. Teachers can evaluate students' conceptual understanding by having them organizing their mathematical thoughts in a written form (Wood, Williams, & McNeal, 2006). Student journals offer a way of *listening* to students communicate within mathematics. Thompson and Rubenstein (2000) suggest using this in conjunction with a class discussion by concluding it with journal writing stems such as the following:

1. Compare what *similar* means in everyday English with what it means in mathematics.
2. Complete the following analogy and explain your thinking: Prism is to pyramid as cylinder is to ______________________.
3. *Square* and *cube* have geometric meanings and are also used for second and third powers. How are geometry and powers related?
4. What is the difference between the square of a number and the square root of a number?

These examples show ways to address some common student confusions using journal writing entries (Thompson & Rubenstein, 2000, p. 571).

Students need to see and discuss writing samples of different qualities to learn what constitutes clear and valid mathematical communication. With a supportive learning atmosphere, students can help each other peer-edit and evaluate each other's writing (Burns, 2004). Checking a classmate's journal for the clarity and validity of a mathematics concept requires a much deeper level of thinking than is required for writing for oneself (Burns, 2004; Chard, n.d.). Another simple strategy that develops mathematical communication is having students fold their paper in half vertically down the middle, solving the problem on the left side, and explaining their thinking on the right side of the paper (Anderson & Little, 2004; Auman, 2008). This helps them see both the computational and conceptual side of mathematics on one piece of paper. Additional activities that may help students become aware of the mathematics around them could be using examples of mathematical terminology from newspapers, periodicals, graphs, or symbols from the media and then writing about what they learned from the presentation or articles (Thompson & Rubenstein, 2000).

Teaching individual words. *Quantum Learning for Teachers* (2006) suggests that teaching students individual words is another way to increase vocabulary. While it is clearly not possible to teach every word every student will ever need for mathematics, it is still necessary to spend time focusing on vocabulary in mathematics instruction. One strategy is to have students explore the concept before vocabulary definitions are provided much like in the strategy of *build concepts first* discussed previously (Network, 2006). For example, students could sort a collection of two-dimensional shapes however they wish. Then, when the students see that it would be helpful to know the mathematical terms (labels), the teacher can build from the students' descriptions of the particular shapes to introduce the formal terms (e.g., square, rectangle, rhombus, triangle, etc.) Experiencing the concept before labeling the concept helps students see why they need to learn the vocabulary (Network, 2006).

Other times, everyday words have broader meanings within the mathematics content which are unknown to students. A strategy for teaching vocabulary directly includes teaching students to read aloud words they already know, then providing a new concept. This is a way of teaching multiple meanings for words students already know (Graves, 2006). For example, if asked, students will say that they know what a tree is. Having students describe what they know about trees, such as explaining the branch and root system, may help introduce a factor *tree* and illustrate how the factors branch out from the original number like the branches of a tree. Using this, they can relate what they know about a tree in nature to what a factor tree is in mathematics.

Vocabulary Review. In *Quantum Learning for Teachers* (Network, 2006) , teachers are taught about the principle of 10-24-7. This is like a combination lock sequence that can *lock* new vocabulary into students' brains and transfer it from short-term to long-term memory. The number *ten* stands for reviewing new vocabulary within ten minutes of learning it. The *24* stands for a quick review of the vocabulary 24 hours after the initial teaching, and the number *seven* refers to a review again within seven days following introduction. Learning new vocabulary and being able to use it correctly takes constant review; therefore, teachers should be consciously using the new vocabulary in subsequent lessons. Reviewing concepts by having students use the new vocabulary orally as well as in writing is essential. By using and encourage students to use the new vocabulary on a daily basis, teachers can more easily work this into an already busy (or overloaded) schedule (Network, 2006).

Specific Strategies for Teaching Mathematical Vocabulary

Keeping journals or logs. When students create ongoing records about what they are doing and learning in mathematics class, they have a chronological record of their learning experiences (Bromley, 2007; Burns, 2004). Burns (2004) suggests that teachers provide the following four prompts to help students focus on their journal writing, especially at the beginning of the year:

1. Using appropriate mathematical vocabulary, write about what we did in class today.
2. Using appropriate mathematical vocabulary, write about what you learned in class today.
3. What are you unsure about, confused by, or wondering about?
4. Describe what was easy and what was difficult for you. (Burns, 2004, p. 2)

Using these journal prompts may help students learn the mathematical vocabulary and may also help them learn how to use the vocabulary correctly (Burns, 2004). More specifically, the journal questions can help the teacher assess which vocabulary their students do and do not understand and why (Burns, 2004). For example, if a student writes "I understand that factors are pieces of numbers, but I don't get why multiples go on forever," the teacher may be able to see that the student has likely confused factors and multiples. This recognition enables the teacher to help the student more effectively.

Word banks. A teacher may want to create word bank charts and hang them up in the classroom for reviewing. In this way teachers can keep word lists for each unit of study and add new words as they appear throughout a unit. These word banks can also include graphics, illustrations, definitions, and examples of the vocabulary in use. The literature suggests that referring to the word banks often will further students' understanding and comprehension of the vocabulary (Bromley, 2007; Burns, 2004; Furner, 2005).

Virtual field trips and mathematical software. The Internet and various computer software programs are now being more effectively used as instructional tools to explore, investigate, problem solve, interact, reflect, reason, communicate, and learn many mathematical concepts. Furner (2005) suggests that teachers can have students take Internet field trips to visit places like zoos and museums and have access to information from NASA and the United Nations to use in mathematics classes. There are many mathematical websites and mathematical software that have been developed to help teachers, students, and parents to keep track of student achievement and record keeping. Some of these also provide learning experiences for students in an exciting and interactive way (Furner, 2005). Exploring with a program like Geometer's Sketchpad® helps students learn and see geometric shapes that are more accurately drawn and certainly drawn quicker than a teacher drawing the geometric shapes on the board and labeling parts. For example, using this software, students can manipulate the shapes and discover what a square has in common with a rhombus. They can also explore the differences as well as similarities of shapes like equilateral triangles, and manipulate the shapes quickly, keeping the characteristics of the shape as they change the size. Students can also see the measurements of the angles and the sides quickly and easily as they change the size of the figures. In this way, students can take an active part instead of being a *watcher* as the teacher introduces mathematical vocabulary.

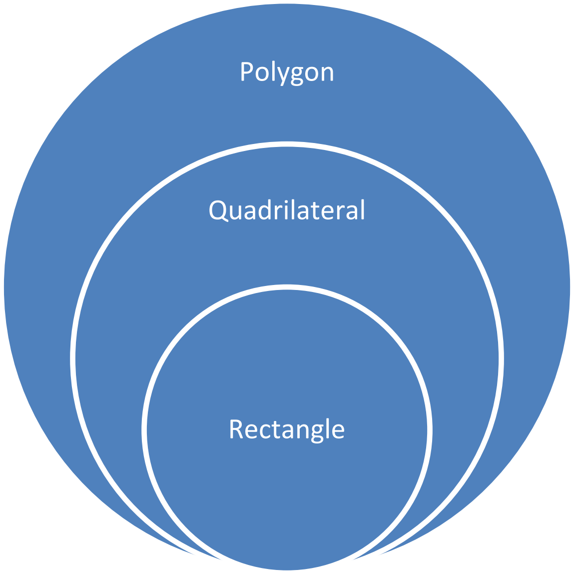
Rich representation of word meanings. In 2000, the National Reading Panel put forth a challenge to educational researchers to develop and document vocabulary teaching strategies. Kucan, et al. (2007) in a yearlong study with high school teachers and their students developed some strategies which were developed specifically for reading, but are relevant for mathematical vocabulary as well. Their vocabulary instruction emphasizes two aspects of effective vocabulary instruction (a) developing rich representations of word meanings and (b) learning about how words work. When students experience multiple exposures to words in a variety of contexts they develop rich representations of word meanings (Barwell, Leung, Morgan, & Street, 2002; Beck, McKeown, & Kucan, 2002; Blachowicz, 2005). Verbal, visual, and dramatic strategies and multiple connections are also important ways to develop students' conceptual understandings. Table 2 provides mathematical examples for developing rich representations. The first column lists the suggestions for multiple rich representations and the second column gives an example of what that type of representation might look like in a mathematics classroom.

Table 2

Vocabulary Instruction Strategies

Rich Representation of Words	Mathematical Examples
Crafting a definition for a word after investigating multiple sources of information	Exploring the meanings of mathematical words in the math text, as well as in online math dictionaries, to craft the students' own meaning of a word.
Locating appropriate synonyms and antonyms	While coming up with a 'student friendly definition' of words and concepts, include synonyms and antonyms of words/concepts. For example: $>$ means 'greater than' Synonym | Antonym More than | Less than $7 > 4$ | $4 < 7$
Constructing visual representations	Drawing figures or shapes to aid with comprehension of meaning.
Role playing	Have students build a geometric shape with their bodies.
Comparing and contrasting	Equation — Expression $8 + 3 = n$ — $8 + 3$ $8 + 3 = 11$ — 11 Have students compare and contrast equations and expressions.
Identifying examples and non-examples	Have students come up with examples of the mathematical terms and concepts in the world around them or non-examples of the term and concepts and explain why it would be a non-example. For example, finding examples of rectangles and then explaining why a rhombus does not qualify as a rectangle.

Rich Representation of Words	Mathematical Examples
Developing analogies	1. Purpose Relationship: Ruler is to line as compass is to circle. 2. Part - Whole Relationship: Ray is to line as arc is to circle. 3. Part - Part Relationship: Vertex is to side as center is to radius. 4. Cause – Effect: 5 is to 25 as 25 is to 625. 5. Position Relationship: Perimeter is to surrounding as area is to interior. 6. Degree Relationship: cm is to km as oz is to ton. 7. Synonym Relationship: Cube is to hexahedron as average is to mean. 8. Antonym Relationship: Parallel is to intersecting as acute is to obtuse. 9. Characteristic Relationship: Square is to rectangle as rhombus is to parallelogram 10. Numerical Relationship: 1/2 is to 50% as 1/3 is to 33 1/3% (Copley, 1989).
Sorting words into categories	For example: Place the words quadrilateral, rectangle, and polygon in the correct circles of this Venn Diagram.



Rich Representation of Words	Mathematical Examples
Specifying situations in which a word might be used	What is the vocabulary word for the numbers in this example? In the example 4^2, we call the 4 the _______, and we call the ^{2} the _______.
Using a word in written and oral discourse	Journaling or exit tickets are examples of written use of mathematical terms. Whole group, small group, and pair-share are all ways to orally use the new terms.

Note. This table was adapted from page 3 of Kucan, Trathen, Straits, Hash, Link, & Miller, 2007.

Learning about how words work. The second part of Kucan, et al.'s, (2007) research describes supporting students by helping them find *units of meaning* through dividing words into parts, such as roots and prefixes. Learning about word histories as well as related words or forms of words also may aid in understanding (Bromley, 2007). See Table 3 for associated mathematical examples. The first column suggests activities that may help students engage in knowing how to figure out what words mean by analyzing either the parts of the words or the context in which it is found in order to make meaning of an unknown word. The second column gives an example of the activity and what it might look like in a mathematics classroom.

Table 3

Vocabulary Instruction Activities

Activities for Learning about How Words Work	Mathematical Examples
Investigating the etymology, or history, of a word	Etymology can help provide a safety net of de-mystification when learning new words or words with multiple meanings. For example, consider studying equiangular or equilateral triangles. Knowing the etymology, you can break these words up into component parts: *equi* (equal), angular, angle, lateral (of a side/sided), and *tri* (3). A three-sided object with all sides equal. It is possible that you will see triangle referred to as trigon. Again, *tri* means 3, and *gon* derives from the Greek word for corner or angle, *gônia*. However, you are more likely to see the word trigonometry -- trigon + the Greek word for measure. Geo-metry is the measure of Gaia (Geo), the Earth (Gill, 2012).
Identifying word roots and meanings	Polygon: ‘poly’ meaning many and ‘gon’ meaning angles or sides (Harper, 2001-2012)
Identifying affixes and meanings	Hexagon: ‘hex’ meaning six and ‘gon’ meaning angles or sides
Generating forms of a word (noun, adjective, verb, adverb)	Noun: multiple, adjective: multiplier, verb: multiply
Generating related words	Thermal, thermometer, thermostat

Note. This table was adapted from page 3 of Kucan, Trathen, Straits, Hash, Link, & Miller, 2007.

Poetry. Donna Hash is a veteran mathematics teacher with more than 15 years experience of using poetry with her tenth grade class. She used examples from *A Joyful Noise: Poems for Two Voices* (Fleischman, 1988) (Fleischman, 1988) and challenged her students to come up with poems for two voices relating mathematics concepts (Kucan et al., 2007). *Poems for Two Voices* can be read by two students with one student reading the text on the right and another reading the text on the left, then both reading the text together when it is directly across from each other. See Table 4 for one example from her students about prime numbers and Table 5 for another example from her students about angles.

Table 4

Prime Numbers

Reader 1	Reader 2
We like to think	
	of ourselves
as the basic numbers.	
2	
	3
5	
	7
No one can	No one can
take us apart	
	factor us further
We're not composite	
We're prime!	We're prime!

Note. Poem from page 4 (Kucan, Trathen, Straits, Hash, Link, & Miller, 2007). This poem is used with permission.

Table 5

Complementary and Supplementary Angles

Reader 1	Reader 2
We are angles	We are angles
My sum is 90 degrees	
	My sum is 180 degrees
I'm complementary	
	I'm supplementary
I make a right angle	
	I make a line
I'm half of supplementary	
	I'm 2 X complementary
We come in pairs	We come in pairs

Note. This poem is from page 5 and is used with permission (Kucan, Trathen, Straits, Hash, Link, & Miller, 2007).

The rule of four. The "rule of four" grew out of the calculus reform movement and the Harvard Calculus Consortium. Hughes-Hallett et al. (1994) in the preface of their 1994 edition of Calculus write: "The Rule of Three: Every topic should be presented geometrically, numerically and algebraically" (p. vii). Vocabulary definitions have emphasized reading, writing, speaking, and listening, so the change was made to "the rule of four" which says that vocabulary should be represented verbally, geometrically, numerically, and algebraically (Foley, n.d.). The examples illustrate algebraic solutions when appropriate and graphical or numerical solutions when an algebraic approach is difficult or impossible to use. Solutions are often supported or confirmed by a second or third approach, helping students see connections across representations. Students are expected to do likewise on homework, tests, project reports, and classroom presentations (Foley, n.d.).

Table 6 provides examples for the Rule of Four. For the term *linear function* there is a verbal description in the second column. Then, the analytical column provides the slope function equation. Next the numerical column presents an equation example of a linear function, and finally the graphical column provides a graph of a linear function.

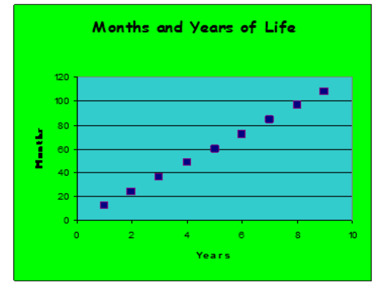
Table 6

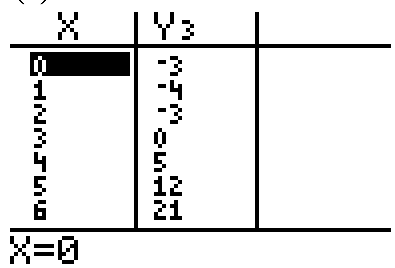
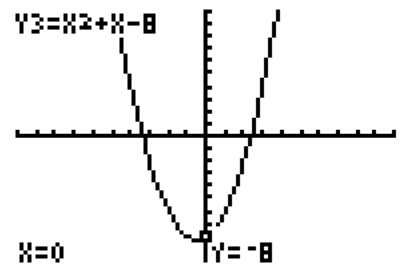
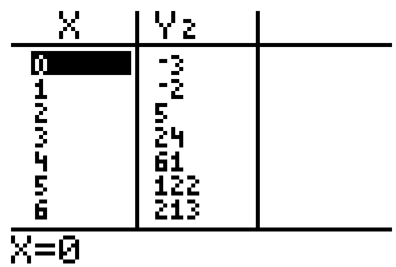
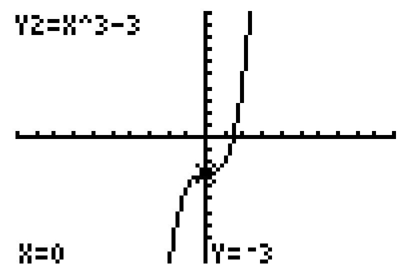
Rule of Four Example

<table>
<tr><th>Term</th><th>Verbal</th><th>Analytical</th><th>Numerical</th><th>Graphical</th></tr>
<tr><td>Linear function</td><td>Any function whose graph is a straight line.

The variables m and b are constant and x is an input variable.</td><td>$f(x) = mx + b$

m=slope

b=y intercept</td><td>f(x) = 12 x + 1</td><td>
</td></tr>
<tr><td>Quadratic function</td><td>A function in which the second differences are constant. The graph is a parabola.</td><td>$f(x)=ax^2 + bx + c$

a, b, c are real numbers and a≠0</td><td>$f(x)=x^2 - 2x - 3$

</td><td>
</td></tr>
<tr><td>Cubic function</td><td>A function with a 3rd degree polynomial.</td><td>$F(x)=ax^3 + bx^2 + cx + d$</td><td>
</td><td>
</td></tr>
</table>

Note: Class example from Michelle Collins, used with permission (Collins, 2012) (Collins, 2012).

Visual Strategies. Much of the media that our students encounter in their out of school lives is visual, so they may benefit when visual strategies are used to support learning vocabulary. Figure 1 shows a sample structured overview related to data analysis. To develop such an overview, students can brainstorm aspects of the topic that they already know. It is also a great summary tool to have students find all related concepts from a unit and place them in the appropriate relationships (Thompson & Rubenstein, 2000). In Figure 1 you can see an example of the connections among and between the words and concepts of statistics. Organizing the concepts into graphic organizers such as this can help students understand the relationships between words and concepts.

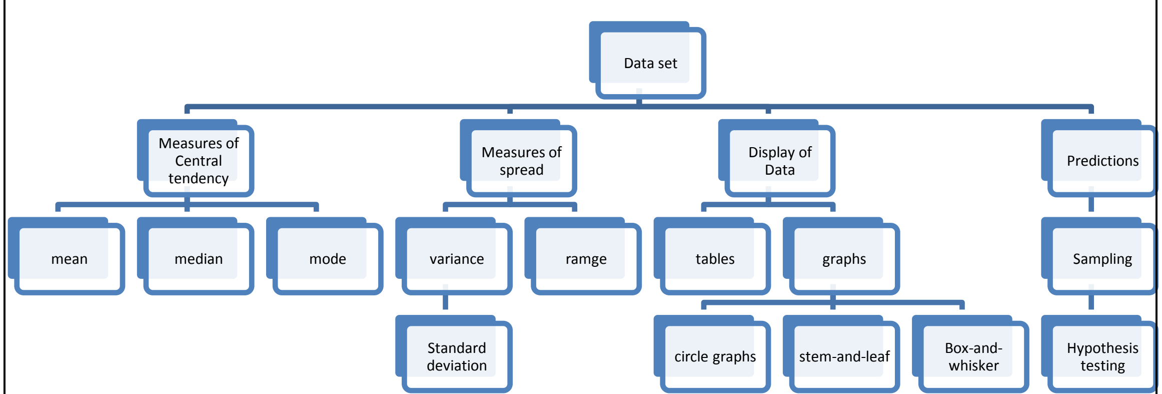


Figure 1. A structured overview in statistics (Thompson & Rubenstein, 2000). Used with permission.

Pictionary®-type games can also be used for review as students draw pictures to try to help their classmates guess mathematical terms or concepts. Playing with words in games or graphics helps students remember the meaning and concepts (Beck, McKeown, & Kucan, 2002). In addition, students can create mathematical cartoons to explain mathematical concepts and teachers can diagnose understanding by evaluating the student's explanation about their cartoon. Mathematical graffiti is another visual tool to aid students in thinking about characteristics of mathematical vocabulary. Figure 2 shows sample graffiti that illustrates various graffiti terminology.

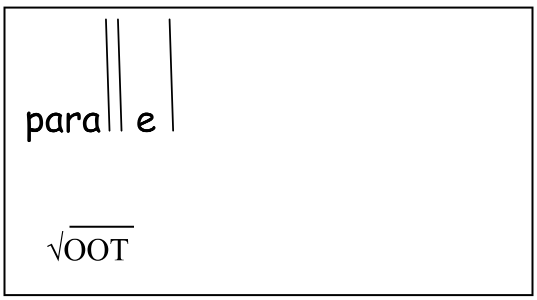


Figure 2. Sample mathematics graffiti (Thompson & Rubenstein, 2000). Used with permission.

Figure 3 shows a Venn diagram comparing mathematical words and the common English words that students may already know. Seeing that the words roots *graph* and *gram* are common to both sets may aid students in learning how the words are connected and yet have different meanings. Students who struggle with written or verbal communication may be more successful if they are provided with these types of artistic opportunities (Thompson & Rubenstein, 2000).

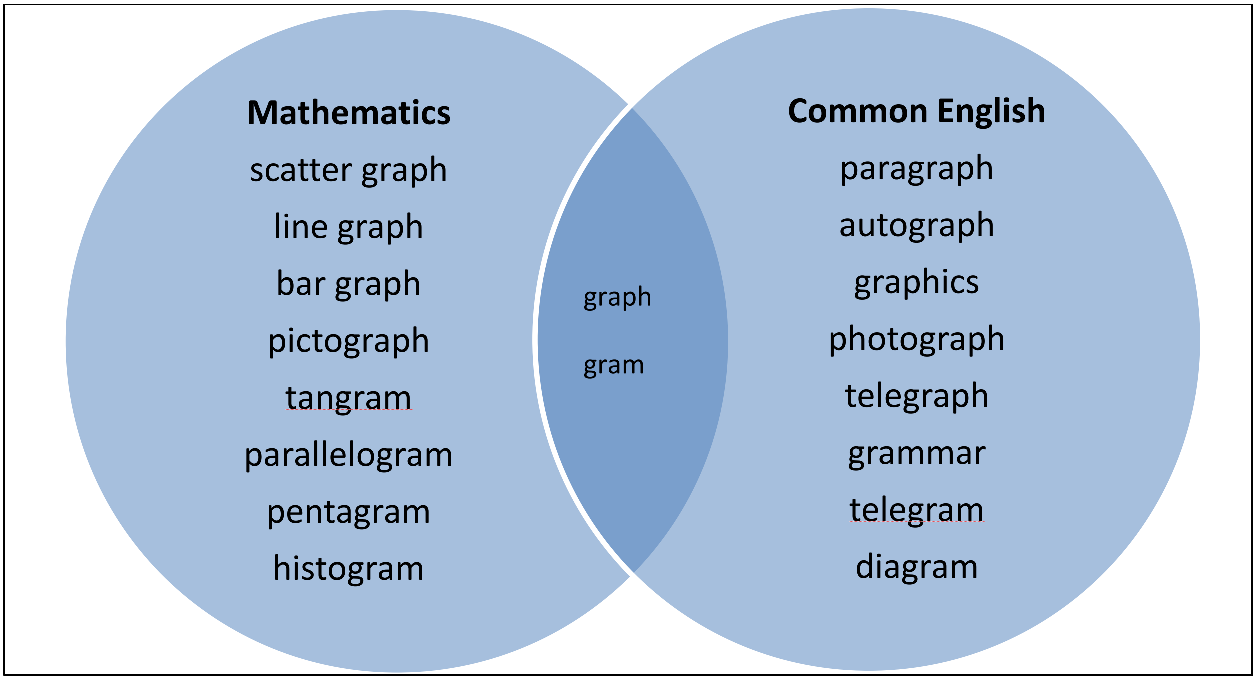


Figure 3. Many common English and mathematics words share the root gram or graph, meaning “to scratch” or “to write” (Thompson & Rubenstein, 2000). Used with permission.

Conclusion. Learning the language of mathematics is crucial to student communication and learning in mathematics. Researching effective strategies for teaching mathematical vocabulary resulted in several ways to help students understand the vocabulary they are expected to learn and use. Such creative strategies may help students understand and remember the mathematical concepts they are taught. In Chapter 3, I will explain some of the strategies I tried in my classroom and my perspective on how those strategies worked for my students. I will also describe some strategies that I hope to utilize in the future.

Chapter 3

Discussion

Sprosty's (2011) examples of "I thought the word meant…" or "I forgot what _____ meant…" and "I didn't know the definition of…" hit home for me as I had heard these comments in my classroom a number of times. Seeing that students may have been able to answer questions like these if they had known the mathematical vocabulary certainly led me to conduct this literature review. I wanted to find effective methods for teaching students mathematical vocabulary. The language of mathematics is a critical factor in students' understanding in mathematics classes (Aiken, 1972; Hersh, 1997; Ogilvy, 1949; Rubenstein, 2007).

It is important to note that several studies show that teaching vocabulary in mathematics is just as important as teaching vocabulary for reading (Chard, n.d.; Montis, 2000; Orton, 1987). However, a major difference between teaching reading and mathematics vocabulary is highlighted in the following example. When students learn new vocabulary for reading, the pictures, context, or previously read information can help students understand, or even guess, what the new vocabulary means (Chard, n.d.). In mathematics, however, much of the time, the pictures, context, or previously read information do not *go with the story* like they do in a reading story or anthology book (Chard, n.d.; Dehaene, 1997). This makes the information less memorable for students; thus the need for a focus on teaching mathematical vocabulary.

Another difference between reading and mathematics vocabulary comes with the opportunity for frequent exposure. As students encounter more text while reading independently, they come across more vocabulary than they may have been introduced to during reading class.

Multiple experiences with mathematical vocabulary are less common, yet essential for learning and remembering that vocabulary (Beck, McKeown, & Kucan, 2002; Stahl, 2003). In a mathematics class, a student may learn the word *diameter*; however, the likelihood of a student ever seeing that particular term while reading for pleasure is very low. In this way, most mathematical vocabulary is not encountered in other settings. Furthermore, if a student encounters a mathematical word outside of school, the meaning may be different than the one the student learned in mathematics class. For example, during a geometry class a student might learn that an obtuse angle is an angle that measures greater than ninety degrees. Then while reading, they may see that an author has described a character in the story as *obtuse*. The student may be very confused if they are trying to picture a character that is larger than ninety degrees instead of a person that is annoyingly insensitive or slow to understand.

In this literature review I have defended the importance of focusing on teaching mathematics vocabulary. I will now re-address my research questions in the following way**:** First, I will discuss some of the effective strategies I found and tried during this literature review as related to my first research question: What are effective strategies for teaching mathematics vocabulary? Second, I will discuss some of the ways the research suggested to teach multiple-meaning words, answering research question two: How might a teacher best teach vocabulary with multiple meanings during mathematics instruction? Third, I will discuss some ways for students to make connections between the mathematical vocabulary and their lives. This is related to my third research question: How can a teacher best help students make connections between mathematical vocabulary and their lives?

Conclusions

Strategies

The research literature suggests many strategies for teaching mathematical vocabulary. In this section, I will discuss some strategies that I found, tried, or would like to try next year in my fifth-grade classroom. In my classroom, I usually have students pair-share or discuss and explain in small groups of three or four students. My students are much more willing to participate in discussion or sharing if they are in small groups. Most of the discussion I refer to in this chapter was done as pairs or in small groups.

Journaling. The strategy that I used the most in my classroom was journaling (Burns, 2004; Burton & Morgan, 2000; Thompson & Rubenstein, 2000; Wood, Williams, & McNeal, 2006). Journaling is an umbrella for many other strategies. Using journaling, I found it easy to integrate the Rule of Four (Hughes-Hallett, 1994) with newly introduced vocabulary words. Most of my mathematics units had about ten to twenty vocabulary words, but some had no new words or very few words. If there were twenty words, there was no way I could address each word to the extent that I may have wanted to, so I had to make decisions about which words I thought students would have trouble with and choose ways to teach those words. When a concept was confusing, I used Burns' (2004) questions to check to see if students had more questions.

1. Using appropriate mathematical vocabulary, write about what we did in class today.
2. Using appropriate mathematical vocabulary, write about what you learned in class today.
3. What are you unsure about, confused by, or wondering about?
4. Describe what was easy and what was difficult for you. (Burns, 2004, p. 2)

If the students could use the vocabulary correctly while writing about what we did and learned in class that day as well as use the vocabulary to describe what they were confused or wondering about, then I knew they understood the vocabulary. If the majority of the class was confused about finding common denominators, then I could address their concerns about the concept the next day as a review or individually if only one person was confused.

Sometimes a student did not have any grasp of the vocabulary and could not answer the questions using the mathematical vocabulary that we had just learned. That showed me, as the teacher, that I needed to take some one-on-one time with that particular student and re-teach the vocabulary or concept. As the teacher, I would much rather know that a student struggled with the vocabulary or concept right after I taught it, instead of realizing so at the end of the unit on the unit test.

I collected the vocabulary journals to check for completion, but I did not grade the Rule of Four component. However, I did give a completion grade for the questions (Burns, 2004) when I was informally assessing concept understanding. In this way, I was able to check for conceptual understanding.

Mathematical graffiti. I did not use the strategy of mathematical graffiti (Thompson & Rubenstein, 2000) except to show examples. The students really liked it and hurried to add it to their mathematical journals. I would like to try and use it in more specific ways in the future. A possible way could be by showing the parallel and square root graffiti and then having them develop their own graffiti for other terms. This may be a good way to help the students remember the words and concepts. I found that if I used the same strategy over and over again, students became bored and frustrated so I tried to use different strategies.

Child-friendly definitions. Having students create child-friendly definitions took time out of my lessons, but after a bit of practice, students could come up with acceptable definitions for their journals. For example, our textbook definition of acute angle is an angle that measures less than 90º (Fuson, 2009) . One of my students had written the definition from the textbook along with a picture of an acute angle. She then had added, “like a little kitten is cute, a little angle is a-cute angle”. In another example, our textbook definition of column is a vertical group of cells in a data table or a group of items arranged vertically in an array (Fuson, 2009). To her formal definition, one child added, “the tall up and down, not the side to side; the side to side are rows.” After having multiple experiences with the new vocabulary, my students seemed to have better success remembering their meanings. Keeping a journal of student representations of new vocabulary helped students, but also helped me informally assess the students’ knowledge, understanding, or misunderstanding of the vocabulary as I collected and read the journal entries.

Writing. When students write about mathematics it makes them organize and make sense of their learning (Burns, 2004). One way that student journaling was valuable to me as a teacher was how it helped me to see where misconceptions, misunderstandings, or incorrect word usage may be occurring. Another aspect I looked for in students’ journals was whether a student added their own definition or reminder along with a picture. If not, I had the student add something to their journal that would help them remember. An example of this type of misunderstanding occurred during a journal entry for *round*. Our textbook definition of round is to find the nearest ten, hundred, thousand, or some other place value (Fuson, 2009). The usual rounding rule is to round up if the next right digit is five or greater and round down if the next right digit is less than five. One student had added this to their journal, “rounding up is like 58 rounds to 60 and rounding down is like 52 rounds to 40". This certainly was a misconception that I wanted to address immediately.

One planning strategy I wished I had utilized was keeping track of the different ways I used journaling. I tried many different strategies and now do not remember which strategies I used with which words. This is something I plan to do in the future. I will do this by putting sticky notes into my book as I introduce the vocabulary with the strategies I used and how they worked. I feel this will not be too time consuming and will help me when introducing the vocabulary the following year. In addition, if there were some really great mathematical graffiti examples, I could put the examples in my teacher edition to be used as examples the following year. I may also put some samples into my own electronic journal and keep track of the strategies used with different words, so I could access these when we change mathematics curriculum in a few years.

Another strategy I used that helped determine if students understood the vocabulary was having students fold their paper in half vertically down the middle, solve the problem on the left side, and then explain their thinking (using mathematical vocabulary) on the right side (Auman, 2008; Thompson & Rubenstein, 2000). I used this strategy with story problems toward the end of the year and it was difficult for my students, but it really made the students stop and think about each step of the problem solving process. Most students wanted to do several steps at once and got confused trying to explain more than one step at a time. Some of my struggling students never did make the explanation match their mathematical work consistently, but most of the class did well after lots of practice. I found an example of a step-by-step guide in my *Step Up to Writing* curriculum (Auman, 2008). See Appendix A. I used this example with my students, having them try a step at a time, then uncovering the step example from the *Step Up to Writing*.

After completing this activity as an introduction, we did several problems as a class as I modeled and discussed ideas that would be the best explanation to go with the mathematical work. Using such a concrete model seemed to help my students, and practicing helped them become proficient at the mathematical explanation of each step. This takes so much time that I did not expect my students to follow this model every time they did a story problem, but I wanted them to be able to use it when they needed it. When the story problem involved several steps, this was when I asked my students to explain each step. They had to slow down and think about each step as well as label and talk about what they found after each step. This took a lot of time, but was worth practicing with my students at least once a week.

I also used a word wall that included the new vocabulary from each unit. I started this because we were required by our principal to post new mathematical vocabulary in our rooms. Eventually, I decided to post really great pictures or examples from the students' journals under the vocabulary on the word wall. As a result of doing this, I saw students' journal entries become better as they hoped I would copy their example or picture and put it on the word wall with the new mathematical vocabulary.

Multi-meaning words. It is challenging for students when mathematics vocabulary has everyday meanings *and* mathematical meanings (Graves, 2006; Thompson & Rubenstein, 2000). A few words, like asymptote, are exclusively found in mathematical contexts, but many others, such as round, base, origin, and expression, have meanings outside of the mathematics classroom. Students may become confused as to the meaning or use that is expected in the mathematics classroom when they immediately think of the general use of the word. When learning the definition for the word *base* in mathematics class, the student may have been *running the bases* at recess just a few minutes before mathematics class. Precise meanings must be very clear to students in order for comprehension to occur (Reehm & Long, 1996).

When encountering multiple-meaning words in my mathematics classes, I found that it was helpful for students to put both meanings in their math journals. In this way, students can see that they are not mistaken when they think they have heard the word before or may know a meaning for the word already. Then, along with a picture and number example for all the known definitions of the vocabulary word, they can distinguish which definition to use in each situation. For example, many students know a meaning for the word *divide*, such as to divide the cookies between friends. This is related to the mathematical definition of divide, which is to split a number into equal groups. Some students may also know the definition of the word divide, as in continental divide, from our social studies class. All of these definitions are related and the meanings are similar, so showing a picture and number example for the different meanings may help students distinguish between the similar definitions.

When a student is encountering a new vocabulary word in mathematics that has more than one mathematical meaning, putting both meanings into their journals also can be helpful. For example, the word *square* may represent in geometry a regular four-sided polygon with ninety degree angles, while another mathematical definition of the word square is multiplying a number by itself, e.g., $5^2 = 5 \times 5 = 25$. By acknowledging both definitions as we learn about them, students have been less confused. One of my favorite moments last year was when one of my students finally made the connection between a geometric square and an algebraic square. He was so excited that I had him come up in front of the class to explain why a 5 x 5 geometric square that had an area of 25 square units was related to the algebraic definition to square a number $5^2 = 5 \times 5 = 25$. He had made a mathematical connection and was so excited about it!

Acknowledging the other meanings of mathematical vocabulary can make a difference to students trying to remember such mathematical vocabulary. If students cannot remember mathematical meanings, reminding them to think of their journals and all the definitions and meanings that they added for that word may spark their memory.

Connections. Making mathematical connections to other school subjects and to real life can help students remember and see the relevance for learning mathematical vocabulary. Seeing how mathematical vocabulary may be used in *real life* nearly always makes remembering the term or concept easier. For example, having the students see how geometry and the Pythagorean Theorem can assist with building a model house that has square corners and straight walls may make a lasting impression.

Helping students make connections between mathematical vocabulary and their lives certainly is a challenge. Of course finding examples all around us, for example right angles within our classroom, was easy to do. It is more difficult to make such connections with more abstract concepts. Sometimes *playing* with the words, such as illustrating a word or concept as in a Pictionary® game, can help students keep that word as their own. My students loved playing Pictionary with our mathematics vocabulary words and we used it as an end-of-the-unit review. For this activity, I would use a list of the mathematical vocabulary that we needed to review. Then, I would have a student come up, secretly show him/her the vocabulary word, and have the student draw a picture that represents that word or concept while the rest of the class tried to guess the word. Trying to draw clues for a word like square root was fairly easy and the students loved that symbol, but trying to draw clues for common denominator took a bit more thinking and more examples before the students could figure out the clues and guess the words.

Practice with the words after learning them is essential in speaking as well as in writing. I had to keep reminding myself while teaching lessons to review the new vocabulary often. This idea of planning for continual review can be difficult and I would like to find a way to make sure I do this.

Finding examples of mathematical vocabulary in newspapers or periodicals, especially using graphs or symbols and then having students explain what the vocabulary represents is real-world use of mathematical skills. Having students tell about how the vocabulary in the articles goes with the graphs is another way of showing students that they will need and use the mathematical vocabulary in their lives, even beyond mathematics class! I did not try this strategy with my class other than discussing the graphs and charts that we found in our Scholastic News® during social studies class, but I think it would be a good strategy to try in future years.

Mathematical websites and software are another way to have students actually experience, explore, investigate, problem solve, interact, reflect, reason, communicate, and learn mathematical concepts (Furner, 2005). Using Geometer's Sketchpad® is a way to keep students' attention and also help them learn. Letting students use Geometer's Sketchpad® to explore was an exciting experience for my class. When building a geometric shape, we could change the size and keep the angles the same if I was showing similar shapes. This was so much easier than trying to draw the shapes on the board and it had so many interactive activities that were more exact than my board drawings. The students could immediately see the connections between the similar figures without waiting for me to draw them.

I also had students use the online activities at the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) to aid in their learning and understanding of mathematical concepts. Having the examples on the Smartboard® in front of the class to introduce adding fractions, and not having to wait for me to draw out the fractions and put them together, kept the students' interest. I also had students explain what they had to do to add fractions using the mathematical vocabulary, such as finding a common denominator by using the least common multiple, adding the numerator, and keeping the denominator the same. Needing the vocabulary to explain the process helped students see the usefulness of learning the mathematical vocabulary. Using the examples from the National Library of Virtual Manipulatives was not as cumbersome as trying to draw them on the board. Later the students were able to log onto the site and try it themselves without having to try to draw the fractions. There are many examples on this site that may help students *see* and understand concepts easier and better than with drawings on the board or in their own journals. Having students work in pairs and explaining adding fractions as they worked helped them use their new-found fraction vocabulary.

After using the fishy fractions site at iknowthat.com (iknowthat.com), my students would ask if we were going to get to play the fishy fractions games as they came into my classroom. This is a game that has a pelican that flies over the water and tries to catch the fish with the right answer on it. There are fish with wrong answers and if the student chooses one of these, the pelican chokes and spits it back into the water. There are several levels. For the first level, the students just match the picture of the fraction to the number representation. The next level is catching equivalent fractions. Future levels include adding and subtracting fractions, matching fractions to decimal equivalences, and fractions to percent equivalences. I am not sure why the students think it is so much fun, but if they are learning, I want them to play the game! As students are playing the game, I like to walk around and ask them questions about what they are learning while playing fishy fractions, helping the students get used to using the mathematical vocabulary to describe what they are practicing. Vocabulary terms and concepts emerged during this process. As an example, students learned that the numerator stands for the number of pieces being talked about and denominator is the number of equal pieces that the whole is being divided into. Finding common denominators, explaining why you only add the numerators and keep the denominator the same, and understanding how to divide a fraction to convert it into a decimal were other concepts they learned. Knowing that they have to explain the process using mathematical vocabulary, I asked for at least four mathematical vocabulary words used correctly in their explanations in order to continue to play the game. Over the last three years, I have decided that students are so used to online games, that it makes sense to use what they know as fun and make it a learning experience.

Limitations

When I started this literature search, I was considering questions that I wanted answered about mathematics vocabulary. I had originally included another question: When and how often should mathematical vocabulary be taught and/or reviewed? As I read information from more than 100 articles and books, I discovered there was no information regarding when and how often mathematical vocabulary should be taught or reviewed. Our school district provided all teachers with a Quantum Learning® professional development in 2006. In that training they suggested the 10-24-7 strategy that I discussed earlier (Network, 2006). However, I did not find any empirical research for this strategy; therefore, this is clearly a topic for further investigation or research.

There was only one reference that conflicted with studies I reviewed about writing in mathematics classes. This study was conducted by Porter and Masinglia (2000) using a group of college freshman calculus students (n= 33). They looked at whether writing could improve the procedural knowledge of students, in other words, did students understand how to solve a problem better because they had written about it first? This study showed that there was no significant difference in procedural knowledge when writing or not writing in calculus mathematics classes (Porter & Masinglia, 2000) (Porter & Masinglia, 2000). So, they concluded that writing in mathematics (during a college entry-level calculus class) did not improve students' procedural knowledge. They did not specifically discuss the vocabulary, just writing to learn in a mathematics classroom. However, their findings about writing are somewhat contrary to other research I read that suggested writing in mathematics classes is helpful (e.g. Anderson & Little, 2004; Bromley, 2007; Burns, 2004; Pugalee, 2001; Thompson & Rubenstein, 2000; among others). It is possible that their research design contributed to their lack of findings. Instead of having the treatment group both discuss and write and the comparison group simply discuss or simply write, they had each do one. This added in an extra variable that could have confounded their results.

Recommendations

There was an incredible amount of research on teaching vocabulary. So much that I had to narrow my topic to mathematical vocabulary. But, in my experience reviewing curriculum materials, I realized that there are no strategies or recommendations specifically for the teaching of mathematics vocabulary in our mathematics textbooks. For instance, how do you help students learn and use the mathematics vocabulary effectively? Although there is a lot of research on this topic, this research does not seem to be reaching the textbooks. Because of this, I would like to see future mathematics curricula developed in such a way that it would aid teachers by making suggestions such as to how to introduce, teach, and review the vocabulary throughout the mathematics textbook.

Teachers need resources that give them direction and implementation strategies for effectively teaching mathematical vocabulary. Curricula that integrate the teaching of the vocabulary into each lesson would save valuable time. In addition, professional development that presents a variety of ideas and creative ways to teach reading vocabulary needs to be made available. Teachers need to learn to be ever conscious of the context of words and be certain students are given immediate feedback and instruction to clarify the use of vocabulary in the mathematics classroom (Kotsopoulos, 2007).

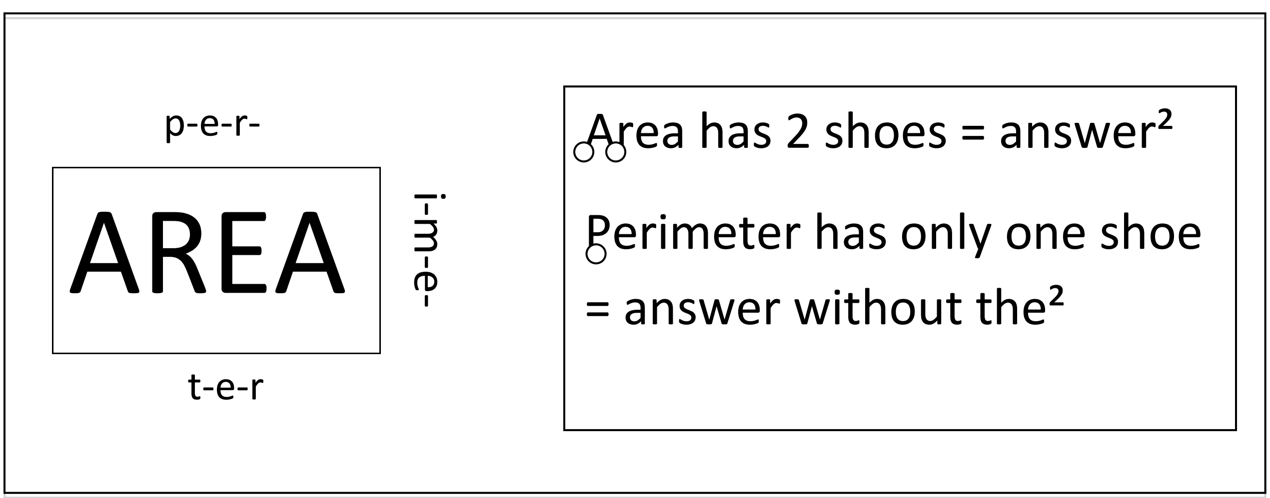
Another possible area for further research may be looking at how to balance the time spent on teaching mathematical vocabulary within the time allotted for mathematics classes. While teaching the vocabulary is essential, it cannot take up a large percentage of the instructional time. Exploring a way to make vocabulary a part of mathematics lessons along with the teaching of concepts and procedures would be a research investigation I would like to see conducted.

I would have liked to have found more research about integrating vocabulary into lessons. Finding a time balance for experiencing, understanding, learning, and remembering mathematical vocabulary is tricky. In the secondary teachers' study of teaching vocabulary (Kucan, et al, 2007), the ten common aspects of teaching vocabulary that the teachers and researchers compiled are all important to consider when teaching mathematical vocabulary to students (see appendix B). For me, the most important of the ten aspects is commitment. Commitment is essential. It takes planning and time during class to teach the new vocabulary as well as review it. If a note to review the vocabulary from the week before was not in my lesson plans, I would not have remembered it. Being willing to experiment with a variety of approaches to teach vocabulary seemed too simple to mention, but I (like everyone) gravitated toward what I was comfortable with instead of going out of my comfort zone and trying a new or different strategy unless I had specifically put it in my lesson plans.

Before starting this research, I thought I did a good job of teaching mathematics vocabulary. After researching so many more ways to teach vocabulary and experimenting with new strategies in my classroom, I think I do a much better job at it. In my fifth grade classroom, I have had my students try many of the mathematical vocabulary strategies and will continue to strive to aid my students in learning the mathematical vocabulary. I certainly have a much larger pool of strategies from which to choose.

I am confident that my students benefitted and will continue to benefit from my new-found knowledge. In the short time since the beginning of this current school year, this has already happened. Two examples (see Figure 4) highlight these benefits. First is a mathematical graffiti example that one of my students created. It shows graphically where the perimeter and area are located on a square. Second, a student who was trying to remember how to label area and perimeter answers, specifically, which answer needs to have squared in the answer and which does not, came up with the idea of putting shoes on the A in Area and the P in Perimeter. She explained to me that if you put shoes on the A of Area, it has two shoes, so the answer is squared. But the P in Perimeter has only one shoe, so you do not square that answer.

Figure 4. Sample mathematics graffiti from my students



While I had tried to give my students many experiences and opportunities to gain the mathematical vocabulary, engaging in this research created a shift. This shift was from me doing and showing the strategies to the students, to them coming up with their own version of the strategies. They became the “doers”, which will ultimately help them remember the vocabulary because it has a better chance of becoming their own to use in reading, writing, and explaining.

Finally, my students became better communicators in our classroom community, whether in small group discussions, journal writings, or whole group discussion. Intentional vocabulary emphasis gave my students the confidence to use or at least try to use the mathematical vocabulary when explaining their thoughts, procedures, or conceptual understanding.

Summary

The research indicates that learning the language of mathematics is a vital tool for student learning (Burns, 2004; Carter & Dean, 2006; Gay, 2002; Hersh, 1997; Kotsopoulos, 2007; Montis, 2000; Orton, 1987). Teaching students the vocabulary, phrasing, and meanings of mathematical language needs specific attention (Kotsopoulos, 2007; Rubenstein, 2007; Thompson & Rubenstein, 2000) . Strategies to make sure that students can talk, read, write, and share their mathematical vocabulary should be part of nearly every mathematical lesson we teach (Burns, 2004; Thompson & Rubenstein, 2000).

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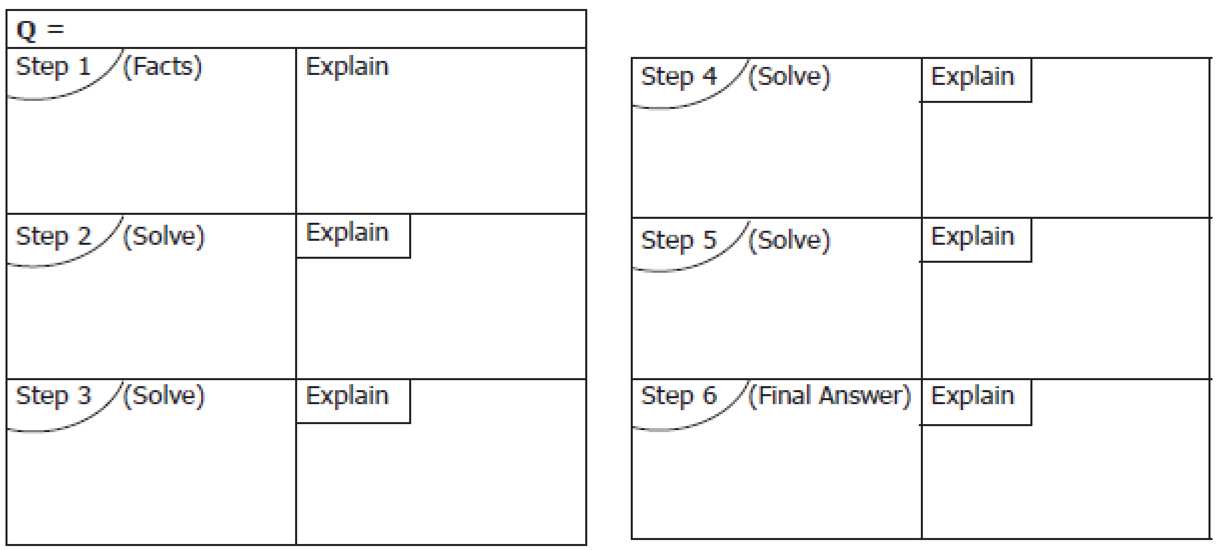
APPENDIX A

Math—Writing to Explain

Try using the following strategy to explain the steps you used to solve a word problem. It will help you think about the major and minor steps that you took to find the answer. The folded paper, the labels, and the two-column guide will make your process visual and easy to explain in a formal paragraph.

1. Fold a piece of notebook paper into six parts. Label the parts as they are in the diagram below.

(Front) (Back)

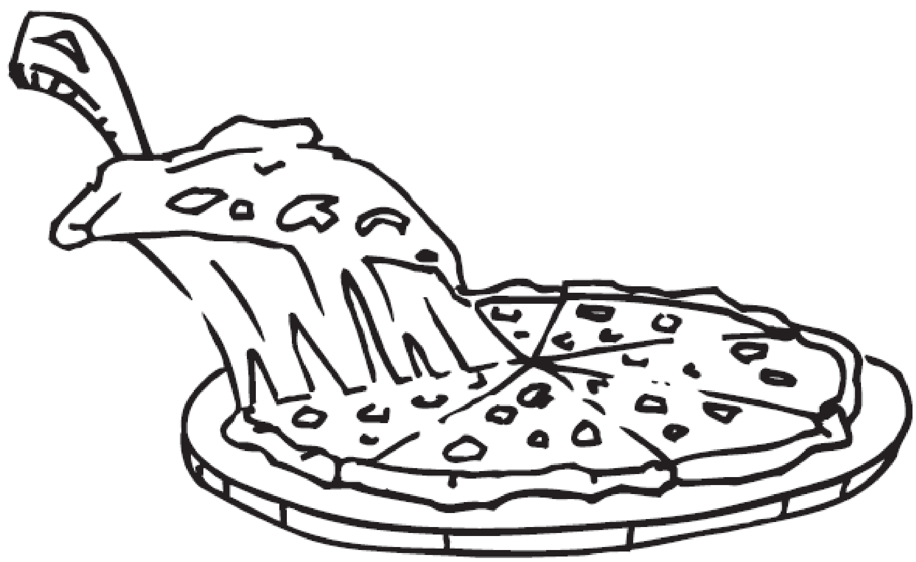


2. The letter *Q* stands for question. Read the math problem. Then rewrite the question next to the letter *Q*.
3. The first step, Facts, reminds you to begin solving any problem by identifying the essential facts.
4. Use Steps 2, 3, 4, and 5 as needed to solve the problem and do the work. Keep in mind that each step indicates a new operation. Each time you start a new step, use a new transition. This will help organize your explanation.
5. Write each step as a complete sentence. Be sure to include appropriate labels. (Examples: feet, square yards, miles, gallons, centimeters)
6. The final step each time you solve a word problem will be the same. Write your final answer in a complete sentence.
7. Write a paragraph by adding a topic sentence that states the problem and rewriting the sentences for each step.

Math—Writing to Explain

Money from Jared

Joe, Julian, Juan, and Jared were hungry for pizza. At Peterstown Pizza Parlor a large pizza with two toppings was $9.45. Juan had $3.50 that he was willing to donate. Julian and Joe each had $2.00. How much money did Jared need to contribute for the boys to be able to buy the pizza?



Math—Writing to Explain

Q = How much money does Jared need?

Step 1 *Facts*	Explain
pizza = \$9.45	First, I listed all of the
Juan = \$3.50	facts about how much
Joe = \$2.00	money the boys had
Julian = \$2.00	and what the pizza
	would cost.

Step 2 *Solve*	Explain
\$2.00	Next, I added \$2.00,
2.00	\$2.00, and \$3.50
+3.50	because I needed
\$7.50	to know the total
	amount of money that
	the boys already had.

Math—Writing to Explain *(continued)*

Step 3 *Solve*	Explain
$9.45	Finally, I subtracted
- 7.50	$7.50 from $9.45. I did
$1.95	this to find out how
	much Jared would need
	to pay. The pizza will cost
	$9.45, and $7.50 is how
	much money the boys
	already have.

Step 4 *Final Answer*	Explain
$1.95	Jared will need $1.95 if
	the boys are going to
	buy the pizza.

Appendix B

Ten Common Aspects for Successfully Teaching Vocabulary to Students

1. Teacher commitment to vocabulary development in terms of planning and class time;

2. Willingness to experiment with a variety of instructional approaches and to adapt those approaches as needed;

3. Setting learning goals in terms of developing rich representations of word meanings as well as an understanding of how words work;

4. Facilitating student access to multiple sources of information;

5. Providing support and encouragement for students to discover connections among words, including forms of words and related words;

6. Giving students opportunities to create multiple representations of words;

7. Highlighting cross-curricular connections;

8. Sustaining commitment to activity-based approaches;

9. Acknowledging the social dimension of classrooms by providing chances for students to work together and to present and perform with and for their peers;

10. Developing interesting assessments involving multiple contexts for focusing on word meanings and features of words (Kucan, et al., 2007, p. 10).