Using Problem Solving to Teach Metacognition in the Sixth Grade Mathematics Classroom

John C. Ripley, III

University of Wyoming

Masters Committee

Linda Hutchison, Associate Professor of Secondary Education, Chair
Michael Morgan, Associate Professor, Early Childhood Mental Health Coordinator
Allen Trent, Elementary and Early Childhood Education

Chapter 1 Introduction

Background and Rationale

During a professional development session several years ago, I was told that formative assessment and feedback were powerful ideas to increase student learning (Hattie, 2009; 2015). As I considered how to use this knowledge effectively to maximize learning in my sixth-grade mathematics classroom I realized that certain mathematics tasks I was already using could provide the platform for both formative assessment and feedback. Furthermore, I realized that for students to use formative assessment or feedback effectively they needed to be capable of some level of self-reflection. This led me to investigate how I could evolve my use of certain math tasks to develop reflective learners capable of high-level mathematical thinking.

Problem Statement

As a sixth-grade mathematics teacher, I work to balance a need to teach procedural processes with developing problem-solving skills using high quality formative math tasks. I have used high quality math tasks for several years but I do not believe I have delivered them in a way that maximizes student learning. I have focused on the specific standards attached to the task and overlooked the problem-solving and self-questioning strategies that I now recognize to be beneficial for students. In this study, a formative math task (FMT), a math task designed to change student thinking, was used to teach metacognition. This was challenging because the standards required that I teach certain procedural processes, but I believe that students can and should learn metacognition skills through the process of problem solving required by FMTs. Metacognition is the ability of a learner to purposefully think about his or her own thinking. I hope this will help my students become reflective learners who are better prepared to face future challenges in education and the real world. As reflective learners, they need to be able to develop the ability to self-question in order to recognize and change mistakes in their mathematical thinking. The purpose of researching this is to potentially benefit students in their pursuit of concept mastery.

Purpose and Research Questions

According to Hattie and Timperely (2007) and Shute (2008) it is necessary to develop reflective learners to increase understanding and use of feedback from formative tasks. Research by Huff and Nietfeld (2008) and National Academy of Sciences, Engineering and Medicine (2018) suggests it is also necessary to teach strategies to develop metacognition skills in learners. Therefore, a teacher must be purposeful in combining formative tasks and metacognition strategies that help students gain confidence in their ability to understand feedback and change mistakes in mathematics. The purpose of this study is to bring these ideas into my 6th grade mathematics classroom with the goal of increasing student learning. The research will be guided by the following questions:

1. Do students report increased use of self-questioning during formative math tasks after practicing metacognition using a post-task survey?
2. Do students report increased use of problem-solving strategies during formative math tasks after practicing metacognition using a post-task survey?
3. What is the relationship between the increase in reported use of self-questioning and the increase in reported use of problem-solving strategies?

Chapter 2 Literature Review

The following assertion guided literature review for this study: the process of solving FMTs can help students learn metacognitive skills, such as self-questioning, that will aid their development as reflective learners. It is organized into three major sections: FMTs, feedback in the problem-solving process, practicing metacognition. These sections include multiple subtopics and are selected to provide a theoretical framework to support the purpose and research questions in this study.

Formative Math Tasks (FMTs)

By choosing to use FMTs in my math classes I am committing to a constructivist classroom where students are engaged in mathematical problem-solving to build their knowledge schema. FMTs are mathematics tasks designed to change student thinking by promoting a *relational mathematics* experience. Inherent in this definition is the importance of thinking and changing thinking. I embrace the notion that "the person who has worked on, and solved, a problem, is not he same person who began working on it" (Schoenfeld, 2018, p. 20). While I do not think it reasonable to present sixth grade students with problems from The Stanford Mathematics Problem Book, I do consider it essential to present them with appropriately leveled tasks that demand similar thinking skills. According to Buck (1959) the value in such problems is that they demand knowledge familiar to the student to be used in a unique or unfamiliar way. In this way FMTs promote problem solving skills and concept mastery beyond procedural processes.

Problem solving

In the preface to the book, *Mathematical Discovery*, the authors use the term "know-how" to describe the ability to solve problems. They are referring to problems "requiring some degree of independence, judgement, originality, creativity" and these are the same type of problems that can be used as FMTs (Polya & Wiley, 1962, p. viii). When I presented an FMT prior to conducting this research, one of my students immediately exclaimed "you have not taught us ratios yet!" even though she could have solved the problem multiple ways by using her understanding of fractions from fourth grade. This is affirmation of the importance of developing problem solving "know-how." I want to teach students to attack a problem using the tools and knowledge that they have.

The three principles of learning and teaching that Polya & Wiley (1962) explain further support the importance of using FMTs to teach mathematics. First, students are engaged in problem solving and spend their time talking to each other or the teacher about the task rather than listening or watching. This is *active learning*; principle one. Principle two is *best motivation* which was summed up by Dan Meyer during a professional development session in Sheridan, WY when he showed us how to "pick a math fight." In the most basic sense this means that you create motivation and buy-in by allowing the problem to create controversy. Having students pick sides during an early stage of the problem-solving process easily accomplishes this. I know I have a good FMT when students quickly and confidently choose a side and my class is split roughly in half. The third principle is *consecutive phases* and this is where the teaching, learning and practicing of metacognition relates to the FMT. I do not design FMTs to be solved in a single session by a routine procedure. Students record their thinking in each problem-solving session and are expected to review previous thinking before moving forward in the next session. This requires students to think about their thinking in each phase of the problem-solving process.

FMTs promote problem solving by presenting a problem that is attainable to students but that requires thinking beyond the scope of procedural knowledge. Problem solving was defined by Schoenfeld as “trying to achieve some outcome, when there was no known method” (Schoenfeld, 1985 in Schoenfeld 2013). This creates a supportive environment for “creative work on an appropriate level” which Polya and Wiley claimed was essential and absent from teachers’ curriculum in the 1960s (1962, p. viii). Based on my experience in teacher preparation courses this type of problem-solving environment is still underrepresented. I attribute this to the challenge of teaching certain procedural skills at a certain time and in a certain order to meet the requirement of specific standards. I also recognize that teaching problem solving is difficult.

Relational mathematics vs instrumental mathematics

The dilemma described above is clearly addressed by Richard Skemp (1987). He uses the terms *relational mathematics* and *instrumental mathematics* to describe different types of teaching and learning. *Instrumental mathematics* is what most teachers are prepared to teach. Skemp attributes this to a number of reasons including that it is usually easier to understand, it delivers immediate and apparent results, and it more quickly leads to the right answer. However, the downfall of *instrumental mathematics* is that it is based on a set of rules and procedures and therefore does not transfer readily to problem-solving tasks. There is a place for *instrumental mathematics* in mathematics education but Skemp argues, and I agree, that it is disproportionate to the amount of attention it receives in the classroom. This aligns with my experience of using high quality FMTs but focusing heavily on specific standards or processes.

FMTs are a means of teaching *relational mathematics* and therefore participants are subject to the inherent advantages. Skemp (1987) identifies the advantage of *relational mathematics* saying it is more adaptable to new tasks, easier to remember, effective as a goal in itself and it’s schemas are organic in quality. Since FMTs present problems with multiple solution paths and unfamiliar contexts they encourage students to employ *relational mathematics* in their problem-solving approach. Skemp and Worboys (1977) note that this shifts the emphasis of classroom instruction from rote learning to intelligent learning and prepares students to use their understanding of content to adapt to unfamiliar situations. Based on the quotation from my student at the start of this section, my students will benefit by shifting focus from *instrumental mathematics* to *relational mathematics* instruction.

Practicing Metacognition

According to Bransford, Brown, and Cocking (1999) the capacity for self-regulation is evident in young children and develops with increased knowledge and experience. Therefore, it is reasonable to expect that students in sixth grade have varying levels of experience and ability to self-regulate. Research published by National Academies of Sciences, Engineering and Medicine (2018) indicates that it is difficult for people to regulate their own learning in formal educational settings. Further, according to Ozsoy & Ataman (2009) and Huff & Nietfeld (2009) metacognitive skills can be taught and learned, placing a significant value on training to improve metacognitive ability. This means that my students are capable of metacognitive awareness but they have varying ability levels, and they will show growth with proper training. In the problem-solving process I modeled and taught self-questioning as a metacognitive strategy.

With the value that metacognitive skills bring to problem solving, the question is not if such skills should be taught but how such skills should be taught. Research by Moos and Ringdal (2012) shows that teachers can foster self-regulation in their students but they need training to do so. What this training should look like is beyond the scope of this project. The significance of the research cited above is to affirm that what I am setting out to do can be done, but that I will have a better chance of being successful with specific training. Additional research indicates that some teachers are better equipped to teach metacognition skills than others. According to Moos and

Ringdal, Michalsky and Kramarski (2008) argue that teachers with more developed metacognitive skills themselves will be better able to foster such skills in their students.

Metacognitive skills and problem solving are closely connected in the context of teaching middle level mathematics because research by Schoenfeld (1985) and Ozsoy & Ataman (2009) has shown that students with higher metacognitive skills perform better in problem solving. Since metacognition can be a useful tool to develop problem solving skills it is suggested that all FMT processes include teaching metacognitive skills in conjunction with problem solving skills and appropriate grade level content.

Feedback in the problem-solving process

By celebrating feedback as a process, I create a mindset among students that learning occurs over time and multiple iterations of a task. If a student is not successful on their first attempt they know that they can use what they learned from the first attempt to try again. This accomplishes two things. First, it encourages students to act upon feedback. There is not the option for students to say “I failed, let’s move on.” Rather, they are given tools to use in the next attempt. They know they’re expected to be successful even if it requires multiple drafts or attempts. Second, because the mindset is established that learning occurs over time, the initial attempt, on which feedback is based, is not a pass/fail situation but a step in the learning progression.

Barnes (2015) makes a compelling argument that feedback should be an ongoing conversation about learning. In his SE2R (summarize, explain, redirect, resubmit) model, feedback takes the form of an online conversation about students’ writing that shows progress from one submission to the next. In his example, success is achieved after submission number three and the ongoing nature of the feedback process shows the student each concept he or she has mastered along the way to the third submission. Celebrating the learning progression not only highlights all of the success the student has achieved along the way, but it also takes the emphasis off of the shortcomings of the first attempt. Part of the way that Barnes cultivates the mindset of learning progression is that he does not assign grades. This way, a student who has much work to do is not facing the frustration, embarrassment or disappointment of a low grade. Instead, he or she is able to focus on the feedback provided and begin making progress toward the learning goal.

Students tend to respond well to feedback that indicates an improvement over time and over past performances. This is logical because whatever the feedback for the current task the recipient still gets confirmation that they are moving in the right direction. Hattie & Timperley (2007) suggest that feedback is more effective "when it builds on changes from previous trials" (p. 85). They also emphasize the importance of goals so feedback that is comparative about past performances should show progress towards those goals. Comparative feedback should focus on the task, the individual and their progress toward the learning goals. It should not measure or rank student work nor should it compare students to one another. By providing comparative (to a student's previous work) feedback you encourage students to focus on mastery rather than 'good enough.' According to a study of medical school students by Harrison, et al. (2016) grades and other comparative rankings promoted a 'good enough' attitude among students and not providing grades actually encouraged students to work toward excellence.

Feedback for FMTs supports learning by guiding students through the problem solving process and by helping them to build self-questioning skills. According to research published by National Academies of Sciences, Engineering and Medicine (2018) research has not yet identified training methods that allow students to transfer self-regulation beyond skills that were directly taught. For this reason I am focusing specifically on teaching self-questioning.

Self-questioning

Self-questioning is a metacognitive skill that can be developed and practiced through teacher questioning. Schoenfeld (2016) states that a problem solving strategy like self-questioning can be broken down to a level that is learnable. “The list” of questions in Polya’s “How to Solve it” is a good starting point (1948). The questions should be posed with the aim of encouraging students to think about themselves and about the process they are using to approach the problem. The questions are triggering metacognitive thinking. In time and with practice these are the same questions that students should be asking and answering themselves.

Schoenfeld (1985) says that asking reflective questions is a method used to teach metacognitive strategy. By modeling reflective questions in the feedback process a teacher can help students practice answering the types of questions that will enable them to reflect on and change their thinking. This is consistent with the idea that feedback should sustain an ongoing dialogue about learning or, specific to this study, a FMT. At various points in the process of solving a FMT the teacher has the opportunity to ask questions such as: What do you think? Why do you think so? How can you prove this? or What about next? Questions such as these challenge students to think about what they have done on the FMT, where they are headed, and what they must do to support their answer. These four questions represent a questioning framework that can model self-questioning when using FMTs with students.

Chapter 3 Methods

Population

Participants were students who assented, and whose parents consented, to participation and included 23 sixth grade mathematics students in a rural intermountain school district. Students attend a junior high school serving approximately 800 students in grades 6-8. The school district is identified as a Title 1 district with approximately 35 percent of students receiving free or reduced lunch. The district serves a community where coal mining is the largest industry. The community can be characterized as predominantly white. Because the University of Wyoming Institutional Review Board required participants to have consent from parents and to have assented to participation themselves, it is possible that participants in this study have a greater commitment and interest in learning mathematics than their peers.

Students learn the Common Core standards for sixth grade and follow the Big Ideas Math curriculum. This curriculum is supplemented with various materials including tasks from the Mathematics Assessment Project and lessons from Illustrative Math. Elementary schools in the district also teach the Common Core standards and primarily follow either the Eureka Math curriculum or Everyday Mathematics curriculum. While most students come from elementary schools within the district there are also students who are new to the district and who may not have experience with the curricula mentioned above.

Instrument

The following instruments were developed to teach self-questioning as a metacognitive strategy. The formative math tasks were designed to present students with a problem in an unfamiliar context and are based on tasks used by Schoenfeld (1985). This requires students to apply relational understanding of mathematical concepts rather than simply repeating a rote process. The post-task survey was designed to gather data from students about their thinking in the problem-solving process, but also to model self-questioning and problem-solving strategies suggested by Polya (1948). By asking students what they thought and what questions they asked, the survey guided them through the metacognitive process of self-questioning. As students work through the problems they should be asking and answering the survey questions of themselves. I'm interested in seeing if they do this as we progress through more problems. The instruments should work together to both measure and teach self-questioning and problem-solving.

Data Collection

Participants completed three FMTs during three separate work sessions in mathematics classes spanning two weeks. Each work session addressed one FMT in the order that they appear in the appendix. In addition to collecting student work at the end of the FMT, data were gathered from anonymous surveys. The surveys contained binary statements and questions to report on self-questioning and problem-solving strategies used in each FMT.

Prior to the start of this study I used the egg box problem to launch the "Interpreting Equations" activity from the Classroom Assessment Project (2015) and it was covered in five to ten minutes of class discussion. In this study, students worked on two similar tasks prior to being presented with the egg box problem. Their work for each task was recorded in a dedicated workspace on a single sheet of paper. This allowed students to see what they had done in their previous efforts to solve similar problems. After attempting each task, students completed the survey in Appendix E. By completing the survey students were practicing self-questioning. I was interested to see if by practicing self-questioning on the survey students would transfer self-questioning and problem-solving skills to future tasks.

Data Analysis

Percentage comparison was chosen as the most appropriate means of data analysis because this research involved 23 cases and sought to generalize participant experience. Identifiers were removed from student data and code numbers were assigned. Percentages were calculated for each research question based on the following categories: decrease, no change, increase. The categories reflect the change in reported use of self-questioning and problem-solving strategies from the first post-task survey to the third post-task survey. The survey included five examples of self-questioning and five examples of problem solving strategies. The study looked at the number of reported examples of self-questioning and the number of reported problem solving strategies for each participant from surveys collected after the first and third tasks. Participants who did not complete a survey for either the first or third task are included in the category "insufficient data."

Chapter 4 Results

Introduction

This chapter discusses the findings of the research. The findings that emerged from the quantitative survey data will be presented according to the three research questions. It should be noted that each of the participants was given a code number to de-identify the data while still allowing quantitative measurement over subsequent iterations of the data collection process. The three research questions that will be addressed in this section include:

1. Do students report increased use of self-questioning during formative math tasks after practicing metacognition using a post-task survey?
2. Do students report increased use of problem solving strategies during formative math tasks after practicing metacognition using a post-task survey?
3. What is the relationship between the increase in reported use of self-questioning and the increase in reported use of problem-solving strategies?

Findings

Findings for Research Question One

Research question one stated: Do students report increased use of self-questioning during formative math tasks after practicing metacognition using a post-task survey?

The quantitative research findings related to this research question will be presented in figure 1 below, and will then be further described and supported.

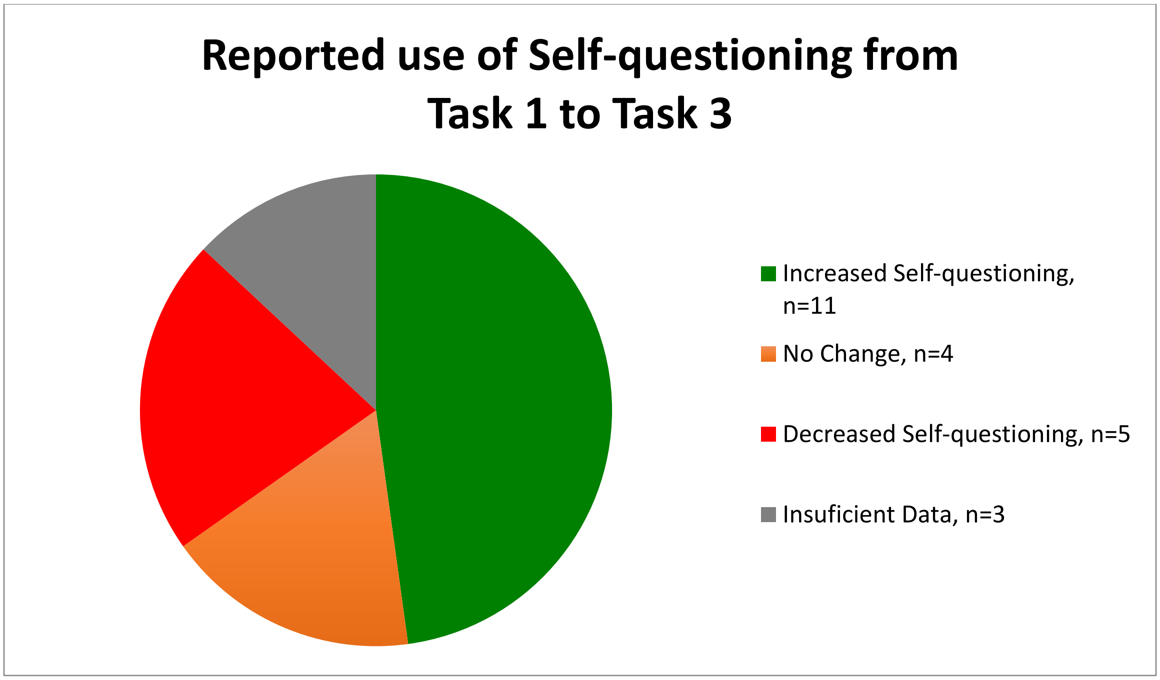

Figure 1. Participants' response to the survey prompt: Please mark each question that you asked yourself while you completed the task.

The survey included five questions that participants might have asked themselves; responses varied from zero to five. The findings show that 48 percent of participants reported increased use of self-questioning by the completion of their third survey compared to 22 percent reporting decreased use of self-questioning and 17 percent reporting no change. Overall, participants reported using more self-questioning strategies after completion of post-task surveys. This suggests that students became more aware of their own thinking by completing these tasks and surveys.

Findings for Research Question Two

Research question two stated: Do students report increased use of problem solving strategies during formative math tasks after practicing metacognition using a post-task survey?

The quantitative research findings related to this research question will be presented in figure 2 below, and will then be further described and supported.

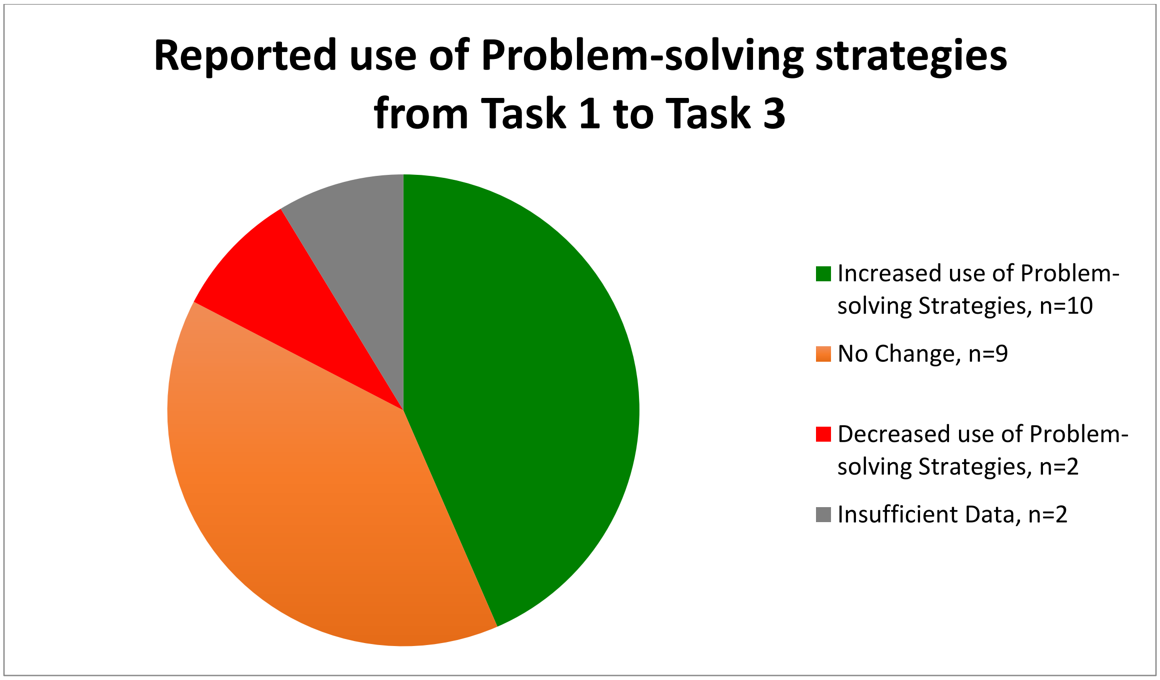

Figure 2. Participants' response to the survey prompt: Please mark each statement that is true based on what you were thinking when you completed the task.

The survey included five strategies that students might have used; responses varied from zero to five. The findings show that 43 percent of participants reported increased use of problem-solving strategies by the completion of their third survey compared to 9 percent reporting decreased use of problem-solving strategies and 39 percent reporting no change. Overall, participants reported using more problem-solving strategies after completion of post-task surveys. This suggests that students became more aware of their problem-solving process by completing these tasks and surveys.

Findings for Research Question 3

Research question three stated: What is the relationship between the increase in reported use of self-questioning and the increase in reported use of problem-solving strategies?

The quantitative research findings related to this research question will be presented in figure 3 below, and will then be further described and supported.

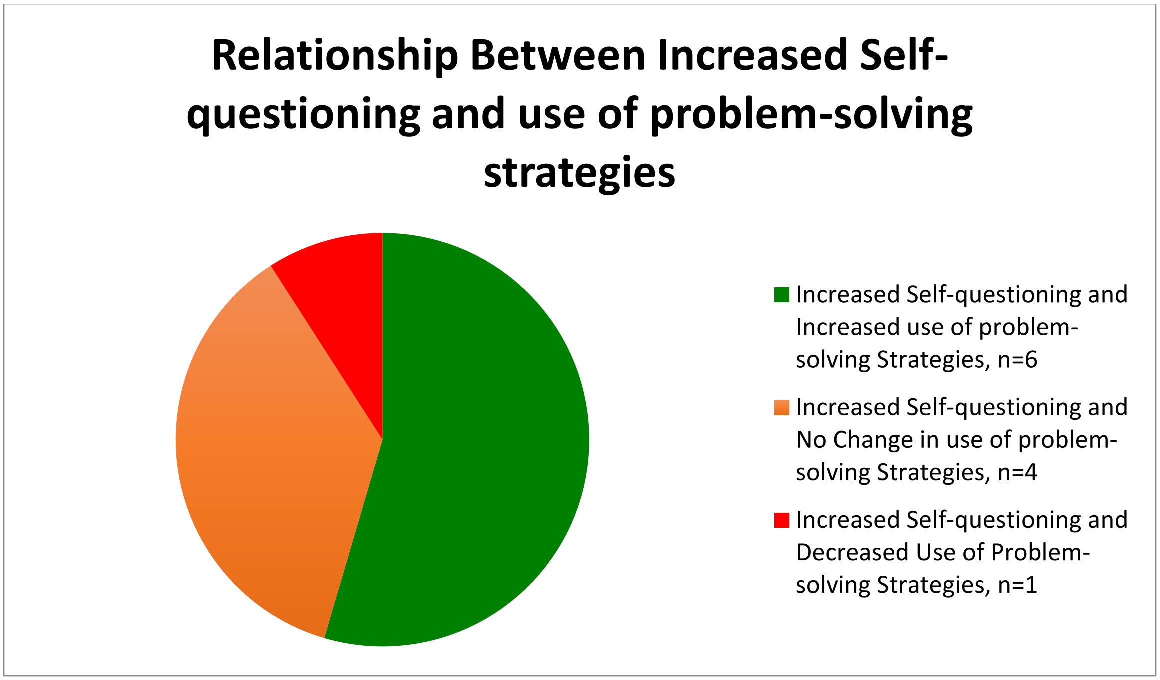

Figure 3. Relationship between increased self-questioning and increased use of problem-solving strategies.

Participants who reported increased use of self-questioning also tended to report increased use of problem-solving strategies from the first survey to completion of the third survey. The findings show that of participants who reported increased use of self-questioning 55 percent also reported increased use of problem-solving strategies compared to 9 percent who reported decreased use of problem-solving strategies and 36 percent who reported no change. This suggests that learning self-questioning strategies may encourage students to apply problem-solving strategies more often.

Chapter 5 Discussion

In analyzing data from the survey, many interesting findings emerged. Self-questioning is a metacognitive strategy that participants reported using. Participants also indicated a relationship between increased use of self-questioning and increased use of problem-solving strategies. This suggests that by teaching self-questioning I can help students develop into more effective problem-solvers. This is important because I want students to develop an understanding of what Skemp (1987) calls *relational mathematics* and to develop the problem-solving "know-how" described by Polya (1962).

Another important discovery from this research is that participants reported increased use of self-questioning and problem-solving strategies with a relatively small investment of instructional time. This suggests that by using quality FMTs and a basic metacognitive tool, like the post-task survey, I can teach skills and strategies that will aid students in developing relational understanding. It is reasonable to expect that using additional formative feedback, particularly feedback that is comparative of past performances, in conjunction with FMTs and metacognitive tools will further enhance student learning. This research was guided by principals of effective formative feedback, such as the decision not to assign grades and the decision to have students show work for subsequent tasks on the same piece of paper, but there is additional opportunity to use formative feedback, such as leading questions that I think will enhance student use of self-questioning and problem-solving strategies.

It is important to point out that the data for this study were gathered over the course of two weeks. Participants reported notable increases in self-questioning and problem-solving strategies in this short amount of time. If FMTs and post-task surveys were incorporated in instruction over a greater period of time it is reasonable to expect greater gains. It is also important to note that by using these instructional tools over a greater period of time students would have additional opportunity to apply relational understanding because there would be a larger context of mathematical vocabulary, skills and procedural knowledge.

Challenges and/or Struggles Experienced

Limitations to this research included having a small sample size, a single trial, and unknown reliability and validity in the survey. The data gathered represents a binary data set and has inherent limitations such as no variation from yes and no answers. In addition, this study is based on self-reported data that is self-evaluative. Research shows that feedback has an observable effect on calibration accuracy and this study did not include explicit feedback to calibrate self-evaluation (Labuhn, Zimmerman & Hasselhorn, 2010). In addition, participants in this study were volunteers whose motivation and interest in the subject do not necessarily reflect the values of the population as a whole. The survey developed for this study was loosely based on questions used in educational research on self-questioning as a metacognitive strategy and mathematical problem solving, however there was no analysis of the reliability or validity of research questions.

Students participating in this study had limited background knowledge of algebraic equations. This was purposeful because it created an unfamiliar context for the problem solvers. However, it also presented challenges in the data collection process. Potential participants may have opted out because of feelings of intimidation or inadequacy. Participants may have struggled to self-report because of unfamiliar vocabulary such as “variable” or unfamiliar mathematical notation such as “6*e*” meaning “6 times *e*.” These challenges could have been mitigated by conducting research over a longer period of time that spanned formal instruction on algebraic equations, algebraic reasoning, and the associated vocabulary. That way potential participants would have more consistent background knowledge.

Future Research

While this research met the primary objective of seeing if students report increased use of problem-solving strategies and self-questioning after completing FMTs and surveys, participants reported more use of problem-solving strategies and self-questioning than their FMT Work Pages showed. Future studies might expand on this research by studying how students can be taught to make their thinking visible.

This study measured the change in reported problem-solving strategies and self-questioning as increased, decreased or unchanged. It did not measure the magnitude of the change nor did it account for the starting and ending points of specific participants. Some participants reported significant increases in self-questioning while others reported modest increase, no increase or regression. Further research is needed to understand how self-questioning can be taught to learners who may not have reported gains through participation in this project.

Another area worth additional research is how self-questioning relates to successful problem solving. This study measured reported self-questioning, reported use of problem-solving strategies, and the relationship between both; it did not measure whether students successfully solved the problems presented to them. Students who are using these strategies but are not successful in correctly solving problems might benefit from exposure to different tools and strategies.

Conclusion

This research supports earlier findings that middle level learners are capable of varying levels of self-regulation (Bransford, Brown and Cocking, 1999). It also supports existing research showing that middle level learners can learn specific metacognitive strategies through reflective questioning (Schoenfeld, 1985). The findings suggest that students who experience growth in their use of self-questioning are also likely to employ additional problem-solving strategies when faced with mathematical problems in an unfamiliar context. This means that I should use FMTs and post-task surveys to teach self-questioning. I should be purposeful about incorporating formative feedback into instruction and I should do this over the course of the school year and throughout all instructional units.

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Appendix A

Formative Math Task- The Table Problem

Suppose there are some tables in a room and that each table has 4 legs.

t = the total number of tables

l = the total number of legs

Chris and Sheila each write an equation to represent the number of tables and the number of legs in the room. Whose equation correctly represents the number of tables and legs? How do you know?

Chris $l = 4 \times t$

Sheila $t = 4 \times l$

Appendix B

Formative Math Task- The Fishing Problem

Suppose that during a fishing trip a person catches 3 fish for every hour that she spends fishing.

h = the total number of hours spent fishing

f = the total number of fish the person catches

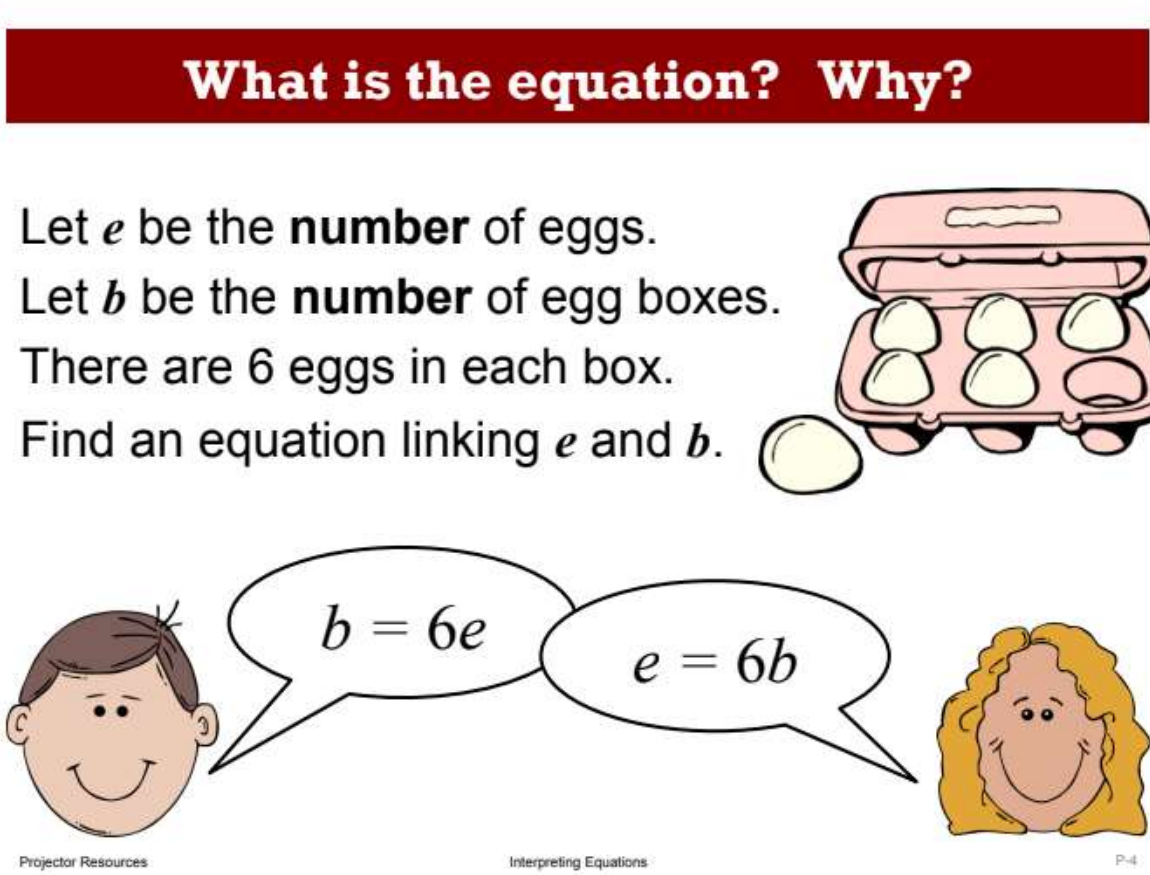
Harold and Stephanie each write an equation to represent the number of hours spent fishing and the number of fish caught on the trip. Whose equation correctly represents the number of hours and the number of fish caught? How do you know?

Harold $h = f \div 3$

Stephanie $h = 3 \times f$

Appendix C

Formative Math Task- The Egg Box Problem



This problem is from The Mathematics Assessment Project.

Appendix D

Formative Math Task Work Page

Interpreting Equations Name:______________________

The Table Problem
The Fishing Problem
The Egg Box Problem

______ By checking this line I want to participate in Mr. Ripley's research study.

Appendix E

Post-Task Survey

Please mark each statement that is true based on what you were thinking when you completed the task. Leave them blank if they are not true.

______ I could see a pattern.

______ I could draw a picture to represent the problem.

______ I could show this information on a graph.

______ I could show this information on a table.

______ With the proper tools, I could construct a model to represent this problem.

Please mark each question that you asked yourself while you completed the task. Leave them blank if you did not ask yourself that question.

______ What do the variables represent?

______ What is the problem asking?

______ What are the possible solutions to the problem?

______ Have I answered all of the questions?

______ Have I seen a problem like this before?

______ By checking this space I want to participate in Mr. Ripley's research study.