

Mathematics Education for
Students with Severe Cognitive Disabilities

By

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Abstract

This literature review covers the current body of research conclusions regarding mathematics for students with severe cognitive disabilities (SCD). It reviews the historical background and importance of mathematics education as a whole in the special education system and for students with SCD. It emphasizes that research in this field is recent, within the past two decades. Although legislation of No Child Left Behind has been a contributing factor, these students are reaching higher potential than ever known possible, and this is calling for a body of research, training, and instructional materials to be available and current. More attention is being given to mathematics education and this attention is for including all special education students. In this paper, standards are reviewed and the cognitive foundations of disabilities are discussed. Curricula and interventions are being developed to meet these needs. Evidence shows an awareness and commitment to making more mathematics available and meaningful for students with severe cognitive disabilities.

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Chapter 1

Introduction

Background

Mathematics is thought to have traditionally begun around 5000 B.C. with the Egyptians and the onset of writing through scribes who would solve simple mathematical problems. The Babylonians also had a foundational influence in mathematics. Mathematics education, however, began much later in the early A.D. centuries, with the Greeks and Romans (Berlinghoff & Gouvea, 2004). Mathematicians were rare and wealthy and could therefore pursue scholarly endeavors. “They built an intellectual tradition that continues to impress everyone who comes in contact with it” (Berlinghoff & Gouvea, 2004, p. 15). Men and women were actually making profound discoveries long before there were names of theorems, postulates, properties, or rules. Observation and use of whole numbers, geometry, and ratios was a natural product of the world around them. Pattern, number, shape, measuring, and record keeping seem intuitive in historical mathematics. Humans have always wondered what, why, and how number and its cohort shape made sense in the world (Berlinghoff & Gouvea, 2004). As mathematics evolved over centuries, discoveries were made that had profound implications and applications. For example, Euclid’s treatise on geometry, *The Elements*, and Newton’s laws of Physics, *Principia* (Berlinghoff & Gouvea, 2004) had a profound impact on world history and development.

Mathematics education has always played an important role in development over centuries. However, mathematics education and instruction for the individuals with extra learning needs (i.e., Mathematical Learning Disability) has not been emphasized (Gersten, Clarke, & Mazzooco, 2007). Dr. David C. Geary defines Mathematical Learning Disability (MLD) as the following:

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A mathematics learning disability would be manifest as a deficit in conceptual or procedural competencies that define the mathematical domain, and these, in theory, would be due to underlying deficits in the central executive or in the information representation or manipulation (i.e. sorting memory) systems of the language or visuo-spatial domains (Geary, 2004, p. 9).

MLD research is increasing. The past 10 years have brought an increase in study, research, and documentation regarding MLD. An added challenge has been students with Significant Cognitive Disabilities (SLD), an even more specified population of individuals who, according to Browder and Spooner (2006):

- (a) require substantial modifications, adaptations, or supports to meaningfully access the grade-level content;
- (b) require intensive individualized instruction in order to acquire and generalize knowledge; and
- (c) are working toward alternate achievement standards for grade-level content (p. xviii)

As result of a recent study by Browder, Harris, Spooner, and Wakeman, (2008), an extensive amount of information and recommendations from 68 experiments on teaching mathematics to individuals with significant cognitive disabilities have recorded information to guide and enhance the mathematics education of students with SCD. Several excellent textbooks: *Teaching Language Arts, Mathematics, & Science to Students with Significant Cognitive Disabilities* (Browder & Spooner, 2006); *Teaching Students with Moderate and Severe Disabilities* (Browder and Spooner, 2011); and *Why Is Mathematics So Hard for Some Children?* (Berch & Mazzocco, 2007) are now informing educators, professionals, parents, and researchers about MLD and mathematics for students with significant cognitive disabilities. The

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field has definitely advanced beyond its infancy, even if it remains a relatively young field (Berch & Mazzocco, 2007). Internet sites for different disabilities are providing families and professionals with outstanding resources to gather, compare, and become knowledgeable and proactive in educating and raising these distinct children. These professional books, articles, and sites are now public domain and have been invaluable in writing this review. More and more information is available. Parents, educators and politicians need to engage in this topic and enter into its discussion and practice (Siegler, 2004). Students with disabilities and their families have capacity to thrive and contribute to society. Mathematic knowledge can only enhance and promote their contributions.

Statement of Problem

Intellectually challenged students have historically been discounted in much of mathematics education of the past. Only in the last decades has research examined the issues relating to MLD, and much is still ill defined and unclear as to what constitutes MLD or dyscalculia (Mazzocco, 2007). A good number of students with cognitive disabilities have MLD. Up until the 1990's, students with severe intellectual challenges have been virtually disregarded in mathematics education except for selected *functional skills*. With the onset of the Individuals with Disabilities Education Improvement Act (IDEA) 1997, educators have become more assertive and aware of including all children in general curriculum (Spooner & Browder, 2006). Students with significant cognitive disabilities represent a population of students with pervasive support and learning needs. They can include those with Down Syndrome, brain injuries, cerebral palsy, autism spectrum disorders, and other various genetic and chromosomal disorders such as Fragile X Syndrome, and Fetal Alcohol Spectrum Disorders.

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Mathematics curricula and interventions follow the ever changing pendulum swings of computational versus conceptual; facts versus problem solving; procedure versus application and meaning; structure versus abstract; and rote mathematics versus making sense of knowledge (Loveless, 2001). Special education mathematics materials have traditionally tended towards the procedural (Browder et al., 2006). Test-driven criteria force special education classrooms into structured curricula and easier data gathering resources, primarily focused on number theory. Studies are suggesting the students with SCD would actually better thrive on a broader, active and inquiry based methodology (Browder et al., 2008). This involves the use of programs that target a broad spectrum of mathematics. While the researchers solve the deepest number theories, pattern driven data, and scientific quandaries in the universe, there is a great deal of learning to be gleaned by these students. There are specific special education and intervention programs and curricula that cover the five strands of mathematics: algebra, data-analysis, geometry, measurement and number, in a broad, hands-on, concrete, questioning, engaging and technologically rich way (Browder et al., 2006). Students with SCD can achieve success and discovery in the mathematical arena. Curricula and interventions need to meet the needs of students with SCD. The development of effective mathematical teaching procedures for the student with significant cognitive disability is presently emerging (Browder et al., 2011). Literature is suggesting and supporting a distinct paradigm for the mathematics students of today, including students with SCD. Trends that promote a balance between procedural and conceptual instruction are developing and progressing (Gersten & Chard, 1999), but it is imperative that correct information is disseminated and professionals are more aware of misconceptions concerning the students with SCD.

Limitations

The research on this particular topic of mathematics education for the students with SCD has been sparse. Students with SCD were primarily institutionalized until the mid 20th century. Yet, this is not the only reason research has been sparse and slow. Mathematics Learning Disabilities (MLD) themselves have only recently become a diagnostic term and recognizable condition. The field was in its infancy in the mid-1990s (Berch & Mazzocco, 2007). There was a small body of research about MLD that was available and even less so for students with SCD.

One other limitation to the study of mathematics has been the capacious amount of research and information studied, analyzed, and written about reading. Research on reading disabilities and its all-encompassing effects on education have stunted that of mathematics (Gersten et al., 2007). The ratio of reading research studies to mathematical studies was 10:0 and 100:1 from 1956 to 1975. It increased to a mere 36:1, 22:1, and 14:1 from 1976 to 2005 (Gersten et al., 2007) (see Figure 1).

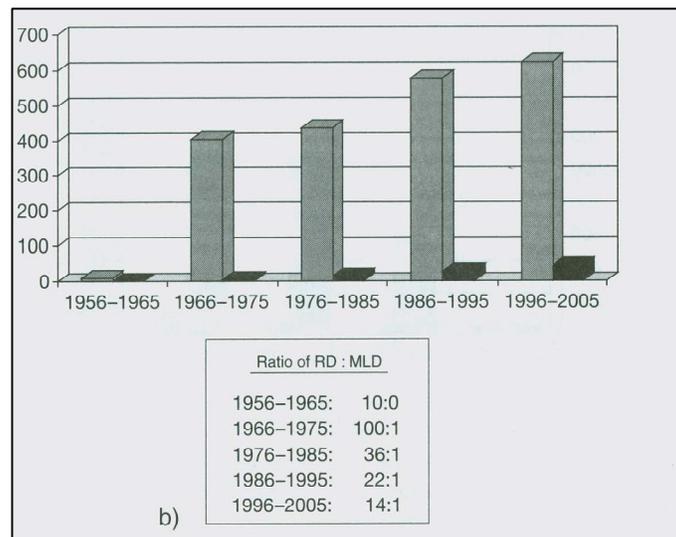


Figure 1. Number of research studies conducted on reading disability (RD) versus mathematical learning disability (MLD) and the ration for each decade from 1956-2005 (Gersten et al., 2007).

Purpose and Research Questions

The purpose of this literature review was to investigate the past and current information about teaching mathematics to students with SCD and their abilities as learners. The past 20 years have produced increased research in this area. The fact that literacy research and reading difficulties received most of the attention in the second half of the 20th century is unquestionable (Gersten et al., 2007). However, the onset of No Child Left behind (NCLB) and the augmentation of the National Council for the Teaching of Mathematics (NCTM) also helped to stimulate interest in mathematics education. Educators, especially special education teachers, and families need to become more informed and competent in mathematics education practice and help these students improve their competency in mathematics. The following chapter reviews current research about mathematics instruction for students with disabilities, generally, and with severe cognitive disabilities, specifically. It will emphasize the what, who, and how we might approach teaching and learning for the SCD student. Finding materials to support these conclusions may be the biggest hurdle.

The primary question addressed in this literature review is:

What teaching practices are most effective for the mathematics education of children with SCD?

Secondary questions include:

1. What is known and documented regarding the mathematic instruction of children with SCD and particularly those with Down Syndrome?
2. Are interventions available and research based to support the education of these students with SCD?

Chapter 2

Literature Review

Before the 1990s, the bulk of mathematic instruction for children with moderate to severe cognitive disabilities (SCD) focused on number skills and was often limited to money and telling time. Mathematics can be challenging to teach and learn, and educators have often limited their instruction for these students to the basics of computation (Browder & Spooner, 2006). The number of research studies in this area for these children is sparse. Although the Individuals with Disabilities Education Act (IDEA), previously known as the Education for All Handicapped Children Act (EHA), was established in the 1970s, it has taken time for the NCTM's principle of equity to filter into the field of mathematics for students with SCD. This past decade, 2000-2011, brought much discussion, research, literature and instructional information regarding mathematics instruction for students with disabilities. Fields of "developmental psychology, cognitive science, mathematics education, special education, and even law have given rise to the multidisciplinary field of Math Learning Disabilities (MLD) research and practice that exists today" (Gersten, et al., 2007, p. 7). Specific mathematical intervention and instruction for the students with SCD was vague if not non-existent prior to recent research. Because of the reauthorization of the IDEA in 1997 and 2004 that state all students with disabilities are required to have access to the general curriculum and to be included in state and district large-scale assessments and the reinforced inclusion of students with disabilities in standards-based reform of No Child Left Behind (NCLB), 2001, all students need to be included in state accountability and assessment systems. "Although federal policy continues to evolve, the changes created by NCLB had a lasting impact on services for the students with severe disabilities (Browder, Spooner, & Wakeman, 2011, p. 40).

The Standards

An important turning point in the assimilation and dissemination of better mathematic education for all students came through the implementation of the National Council of Teachers of Mathematics (NCTM). Established in 1920, the NCTM has become a highly respected and influential voice for mathematics. This organization is by definition “*a public voice of mathematics education, supporting teachers to ensure **equitable mathematics** learning of the highest quality for all students through vision, leadership, professional development, and research*” (NCTM, 2011, p. 1). The International community has also influenced these standards.

NCTM has provided professional guidance through mathematic teachers and others knowledgeable and experienced in mathematics. Prior to the 1980s, curricular content came primarily from textbook publishers and standardized test publishers (NCTM, 2011). They published three landmark documents: *Curriculum and Evaluation Standards for School Mathematics (1989)*, *Professional Standards for Teaching Mathematics (1981)*, and *Assessment Standards for School Mathematics (1995)*. NCTM was the first to introduce “standards” in 1989. In addition, states each created their own Standards documents. In the late 90’s, other organizations (e.g. The America Diploma Project, Performance Standards, and College Board Standards for College Success) began producing Standards documents as well (NCSM, 2011).

Using the input of mathematicians, teachers, and researchers, the *Standards 2000 Project* came to guide the teaching of mathematics. Standards were identified that centered on following six principles: (a) Equity, (b) Curriculum, (c) Teaching, (d) Learning, (e) Assessment, and (d) Technology. Additional standards were developed that included five content areas: (a) Number and Operations, (b) Algebra, (c) Geometry, (d) Measurement, and (e) Data Analysis and Probability. Also, five process areas were identified: (a) Problem Solving, (b) Reasoning and

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Proof, (c) Communication, (d) Connections, and (e) Representation (NCTM, 2011). In 2009, the National Governors Association and Council of Chief State School Officers decided that it was time to address the incoherence of each state having their own standards. Subsequently, they commissioned *Achieve*, a bipartisan, non-profit education reform organization, the *College Board*, and *ACT* to develop a set of college and career readiness standards that would be available for states to adopt. Jason Zimba, William McCallum, and Phil Daro were mathematic experts who also had enormous input into these refined standards. Daro then chose other renowned education researchers, educators, and policy makers with specialty areas to help lead the standards development process (M. Busch, personal communication, December 6, 2011).

More specific grade level standards were developed based on The College and Career Readiness Standards. These were voluntary national standards. National organizations such as NCTM and NCSM and experts provided feedback for revisions. As of 2010, states were systematically adopting this unifying set of standards called the Common Core State Standards. While 44 states have adopted these standards, 48 had input in the writing process (M. Busch, personal communication, December 6, 2011).

Mathematics education will likely eventually be guided by these CCSS. NCLB has legally mandated that students with disabilities be included in state testing and be given access to general curriculum (NCLB, 2002). State testing will be driven by the Common Core Standards. This, however, is where it gets tricky in terms of the inclusion of special education students. Special educators have long been challenged to teach students with extremely varied needs and provide an individualized education for each of them according to their legally binding Individual Education Plans (IEPs). In addition, this is one of the most transient sectors of education. Special Education teachers are in high demand and transition often (McLeskey, Tyler,

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& Flippen, 2004). Experts in special education are difficult to find (Smith et al., 2011). Because of this, the binding standards of NCTM and IDEA are helpful and difficult. Special educators and special education systems are held to a high standard, yet a serious shortage of qualified and knowledgeable professionals exacerbates the problem of quality education for students with SCD. Mathematics education needs to be broadened to the previously mentioned content areas, principles, and processes, and this is particularly important to address in special education and for the students with SCD. IEP teams rarely know what opportunities a student may have as an adult. To limit or restrict a student's education based on assumptions about the student's disability would be unfortunate (Browder et al., 2011).

One reason for the lack of research on mathematic disability (Gersten et al., 2007) was the assumption that students with SCD lacked the ability to learn mathematical concepts (Browder, Spooner, & Trela, 2011). This misconception has been challenged in recent years. Although students with SCD have a wide variation in their abilities to learn, retain, and use knowledge, including mathematics, they need the opportunity to discover their capabilities like all other students. These students deserve as equitable an education as possible. In mathematics, this includes the opportunity to learn and reason through all the content areas and all the process areas.

Five Strands Broader Base

These 2001 standards emphasize five content areas including: (a) algebra, the study of patterns, relations, and functions; (b) geometry, the study of spatial organization; (c) data analysis, the study of organizing and interpreting facts and data; (d) measurement, the study of defining attributes in the standard format; and (e) numbers and operations, or the study of quantity and number (Browder et al., 2011). Students with SCD may be able to master more complex mathematical concepts than once considered possible (Jimenez et al., 2008). These

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skills will then prepare them for more educational, job, and life opportunities in their futures. More and more SCD students have opportunities to go to collegiate programs such as “Think College” and “Learning into Future Environments.” Mathematical skill creates better problem solvers, communicators, and thoughtful adults. Discovering a broad and useful facet of each of these content strands is not only a part of NCTM’s standards but a balanced and relevant mathematical instructional program for all students including those with SCD.

Algebra. The study of Algebra is often portrayed as a high level area involving only complex functions, variables, and slope. However, the underlying principle of Algebra is the discovery and recognition of pattern and its relationships. Recognizing the sequence of units of pattern such as blue, blue, and red or 3, 6, 9, and 12 are all foundational to algebra. Being able to determine the next unit in a sequence can lead to all types of observations in nature. For example, in machinery, poetry, and music the patterns involved can be exquisite. Changes involved in our world such as temperature, finances, time, and goals are the essence of the graphing involved in slope and algebraic understanding. Being able to recognize algebraic slope in the real world does not necessarily involve knowing or manipulating the formula. Students with SCD can certainly gain skills and relevance in algebraic problems and observations in the real world (Bird & Buckley, 2001).

Geometry. Geometry involves recognition of shape and spatial relationship as well as coordinate planes, directional terms, and maps. From the foundational terms of point, line, curve, and shape, to the complex geometry of Euclid, there is a great deal of geometry to be gleaned at all developmental levels. Concepts of perimeter and area are very simple and useful. Students with SCD will relish the four quadrant coordinate plane as they plot points and discover points of intersection. Mapping skills, grids, and directions across town are also geometric in nature. Some

key job opportunities require knowledge of this concept. Geometry skills provide descriptions of objects and moving through space (Browder et al., 2011).

Measurement. Being able to use money and tell time are such basic life skills that they are a regular addition to most IEPs. However, measurement as a mathematical topic is so much more. Terms such as large, slow, and deep are foundational to daily life. Tools that measure length, weight, and capacity are important in classrooms and homes. Designs that involve angles and curves are a part of the world around us. Many students, including those with disabilities are curious and naturally want to measure their world (Ginsburg & Pappas, 2007).

Data analysis. Data analysis is about organizing information and answering questions about data. Students with SCD have long been involved in managing their behavior and academics and creating graphs or charts to do this (Browder et al., 2006). Though maybe not taught as data analysis, these children and adults are already adept in graphing and charting skills. Students can learn to apply these skills elsewhere and make great sense of and interpret data. Charts, tables, and pictures are excellent ways of modifying math to be better understood. Graphing and charting information gives mathematics a visual aspect and refreshing application of quantitative data in a variety of subjects such as comparing sales, monitoring change over years, or knowing statistics of products on the market.

Number and operations. Numbers and operations are the most obvious aspects of mathematics. Representing and computing numbers is seen every day as we make purchases and move from one place to another. Relationships among numbers and numbers systems literally run our world. From speed signs to percentage sales and taxes or from government to the mother buying diapers, numbers are as intrinsic in our lives as letters. In early years of education, particularly for the students with SCD, teachers and students often got bogged down with money

recognition and making change. These are actually higher level thinking skills for all students and many kindergarten and first grade students have great difficulty with them. There is so much more to number than representation, recognition, counting, and operation work. Many students with SCD are capable of identifying and understanding numbers and combinations of numbers, number correspondence, place value and numerical order, even fractions. Students can learn to apply entry-level number sense in increasingly complex contexts across the grade-level content of mathematics. They can progress in sophisticated number use across the grade levels. Computer software, calculators, and other materials may be used to manage difficult facts and larger numbers (Browder et al., 2006).

Number sense. Number sense is a different skill set than number and operations. Number sense refers to a student's fluidity and flexibility with numbers. The NCTM *process standards* flow out of the foundation of number sense, making sense of numbers, what they mean, mental math, and real world application. Constructive, conceptual mathematical thinking is founded on this number sense. Phonemic awareness is the building block to reading, and number sense is the same to mathematics (Gersten, 1999). Developing such a sense is difficult for the student with SCD but not impossible. "The most important outcome students can receive from mathematics instruction is to learn to solve problems" (Browder et al., 2011 p. 175). We must direct our attention towards developing this sense.

Focus on a broader base of mathematics rather than a deep, detailed understanding of the intricacies of math is proving to be a valid approach, specifically for students with SCD. Additional research is being conducted and supporting instruction that covers all strands of math. "There is widespread agreement that children should be exposed to a broad range of mathematical content" (Ginsburg & Pappas, 2007, p. 434). Conclusive research from 40 editors

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of *Why is Math so Hard for Some Children* indicates that one major goal of “mathematics education is to promote mathematical thinking and communication, not just the mastery of skills and concepts and this kind of intervention may be especially useful for the students with MLD (p. 435)” and SCD. Studies suggest that math instruction needs to:

1. Focus on the big ideas-generalizable concepts rather than individual details,
2. Teach conspicuous strategies, neither too broad nor too specific for conducting math operations and solving problems,
3. Make efficient use of time in prioritized objectives,
4. Communicate strategies in a clear and explicit manner, and
5. Provide practice and review to promote retention (Carnine, 1997).

Table 1

Sample Individual Education Plan Goals

Strand	IEP Goal
Algebra	When given two or more shapes, numbers, colors or designs, student will be able to complete the pattern identifying the next shape in the pattern, increasing from 20% to 80% as measured by teacher created assessments
Geometry/ Measurement	When given a triangle, rectangle or square, student will be able to measure the sides of the shape and use the measurements to find the perimeter of the shape using the appropriate formula. Student will also find the measurements of perimeter using various units of measurement (centimeters, inches, and feet) up to twenty improving from 0 out of 10 correct to 7 out of 10 correct as measured by data collection and teacher observation.
Data Analysis	When given various sets of data to be analyzed and compared, student will graph the information using various types of graphs (line graph, pie charts and bar graphs), improving from 20% to 80% accuracy as measured by teacher observation and data collection.
Number and Operations	When given a fourth grade level addition, subtraction, division, and/or multiplication problem, student will input the digits into a calculator and solve the equation, improving from 20% to 80% accuracy as measured by data collection and teacher observation

Note. Sample IEP goals that cover all strands of mathematics

Cognition

Genetic, traumatic, and cognitive obstacles combine to cause significant delays for students with SCD. Students with SCD often have similar deficits and strengths (Barnes, Fletcher, & Cobbs, 2007). Cognitive processing involves the mental operations we do to evoke meaning or solve a problem whether it is word related or number related. Cognition is how we acquire, organize, represent and process knowledge visually, intuitively, selectively and through reasoning. It is our thinking and our remembering (Pellegrino, Chudowsky, & Glaser, 2001).

Whether a student has traumatic brain injury or is born with cognitive disability, it manifests itself in a delay of processing information and difficulty remembering certain aspects

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of number theory and spatial operation. This review will focus on students with SCD cognitive processing problems in learning of mathematics, particularly those with Down Syndrome.

Unfortunately, “professionals have not given a great deal of thought to how students with severe cognitive disabilities think” (Kleinert, Browder, & Reeves, 2009, p. 305). The focus has been on behavior modification and analysis that lead to more independent living skills. Applying cognitive theories to education of students with SCD has been ineffective primarily because they focus on the deficit or what these students lacked instead of a “capacity building-model approach” (p. 305), which emphasizes what students *could* do. Without a cognitive framework such as Piaget’s 1978 model or more recent constructs of James Pellegrino’s cognition vertex (Pellegrino et al., 2001), understanding how these students think and construct knowledge is “especially difficult and great care must be taken in making inferences about these students cognition” (Kleinert et al., 2009, p. 306). A model developed by Pellegrino (2001) suggests “four perspectives for understanding the nature of human learning and knowing. Each of these perspectives has important implications for the assessment of students with SCD” (Kleinert et al., 2009, p. 306).

Pellegrino’s (2001) *differential perspective* emphasizes products, learning about how *much* we know. Assessment scores play a huge role here and have been primarily based on mental age versus grade level production. Students with SCD have scored poorly over the years and have at times even been excluded as untestable. This perspective severely limits the understanding of what these students are capable of performing. The *behaviorist perspective* regards task-analysis and stimulus-response associations. Its focus is systematic assessment particular to measurable and observable targeted skills, a small bit of *what* we know. This has been the most influential perspective regarding education of the students with SCD but does not

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focus on “*how* students construct, organize and/or use the knowledge they attain” (Kleinert et al., 2009, p. 307). The *cognitive perspective* focuses on this *how*. It emphasizes strategies to connect knowledge to quality, organization, and meaning. It is more about growth over time, a much more essential goal for students with SCD. “A one-time snapshot of what they know might not capture the significant gains in how they have learned to represent their knowledge over time” (Kleinert et al., 2009, p. 308).

The fourth and final model of learning and knowing according to Pellegrino (2001) is the *situative*, or socio-cultural, *perspective*, one’s place in a community of learners. Here, controversial yet supported research practices such as inclusion and least restrictive environment issues arise. Students use their skills for “real work of community”. Inclusive settings “promote not only social benefits but attainment of educational goals” (Kleinert et al., 2009, p. 308). Students with SCD may have trouble transferring skills to contribute to the real world and should be allowed practice and effective strategies to develop application skills (Kleinert et al., 2009). Finally, given the current emphasis on technical competence, which draws on high levels of mathematics, mathematic ability and achievement, seem critical (Swanson, 2007).

There has been considerable research on the neurological factors associated with intellectually disabilities. Suffice it to say, little conclusion can be drawn as to the neurobiological causes of Mathematical Learning Disability (MLD). Much research is ongoing but conclusions are unsubstantiated at this point (Geary et al., 2007). Memory deficits, however, represent a significant problem. “It has been well established that many children with MLD do not perform as well as their same-age peers on a variety of working memory tasks”(Geary et al., 2007). Memory issues in students with SCD should be recognized in order to develop an overall

understanding and plan for their teaching and learning. Working memory and long term memory are the fundamental elements of cognition and the mind's cognitive architecture (Pellegrino et al. 2001).

Working memory. Working memory is sometimes referred to as short-term memory. It is what people use to access and process information right in front of them. Its key component is capacity (Pellegrino, Chudowsky, & Glaser, 2001). Working memory is ability to hold a mental representation in mind while simultaneously engaged in other mental processes (Geary et al., 2007). It is a “highly limited system” (Pellegrino et al., 2001). Alan Baddeley's model of working memory (1986) proposes three looped components of memory. These include:

1. Central executive- component controlling attention and processing mental information
2. Visuo-spatial short-term memory, holds images, pictures, and locations; and
3. Verbal short-term memory or phonological loop, holds verbal words and numbers typically speech based (Broadley, MacDonald, & Buckley, 1995) (see Figure 2).

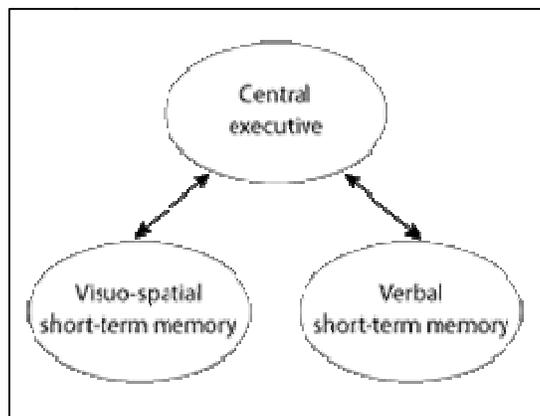


Figure 2. Working Memory System (Buckley, 2008)

Typically, the older we get, the better our working memory because of the faster loop and the more capability. Students with SCD need learning strategies to chunk or code information, as

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their working memory capacity is weaker and they have trouble with multistep directions (Kleinert et al., 2009, p. 309). Working memory certainly affects long-term memory but “what matters most in most situations is how well one can evoke the knowledge stored in long-term memory and use it to reason efficiently about current information and problems” (Pellegrino et al., 2001, p. 3).

A “virtually limitless store of knowledge” (Pellegrino et al., 2001, p. 3), long-term memory stores two types of information: semantic, the way the world is, and procedural, how things are done. Memory skills are particularly crucial for students with SCD (Kleinert et al., 2009). Students must connect the ways of the world to the procedures of the world, especially in regard to mathematical information. “Procedural knowledge can be gained only by sustained, explicit practice across the range of exemplars or situations in which the student would be expected to learn the skill” (Kleinert et al., 2009, p. 309). Difficulties in number procedures and basic fact memorizing by students with SCD stem from difficulty storing and retrieving this information in long-term memory. These foundational skills inhibit other sequential mathematical skills. The deficits in memory are to be acknowledged and recognized, and “identification of the underlying processes that might account for these deficits could provide a clue to either remediation or alternative teaching and learning strategies” (Broadley et al., 1995, p. 3).

Finally, Pellegrino (2001) also makes a distinction between concepts of development and learning. Some types of knowledge, particularly regarding mathematics, come in the course of normal development while other types are learned through deliberate teaching. Geary (2007) calls these “biologically primary cognitive abilities” and “secondary cognitive abilities”. The first are fairly innate in our development, the second require effort. “Yet for students with SCD,

these primary cognitive forms of learning are often not acquired incidentally, but rather require very intentional and focused instruction as well”(Kleinert et al., 2009, p. 311).

Visual, auditory, or kinesthetic (VAK) learners. Students with SCD have difficulty acquiring information through auditory means (Horstmeier, 2004). “Verbal short-term memory tasks typically involve auditory presentation of the to-be-remembered items, and clearly if one has difficulty in hearing and identifying these items then one's memory for them is likely to be severely reduced” (Jarrod & Baddeley, 2001, p. 18). Hearing difficulties are known to be relatively common in SCD students such as those with Down Syndrome (Jarrod & Baddeley, 2001). Students with SCD do have “higher incidence of sensory or physical disabilities than students with less severe disabilities” (Kleinert et al., 2009, p. 311), and this affects their kinesthetic or motor abilities and learning through imitative or exploratory hands-on methods.

Although visual-spatial acquisition of information may be weaker for the child with SCD than a traditional child, research suggests visual and kinesthetic teaching materials are still a stronger place to implement learning than auditory methods (Horstmeier, 2004). Special instruction and practice is needed and, fortunately, information and practical help are emerging.

Research studies consistently show that children such as those with Down Syndrome have characteristically poor short-term memory. Children and adults with Syndrome show poor auditory memory compared with visual memory and recognition memory (Broadley et al., 1995). The Down Syndrome child has difficulty with the abstract ideas of math. Arithmetic and number skills are areas of particular difficulty for individuals with Down Syndrome. “Children with Down Syndrome show severe difficulties in mastering basic number skills as assessed by tasks that include size and numerosity judgments, counting and simple arithmetic” (Brigstocke et al., 2008, p. 75). Number vocabulary is difficult, as it is with many children since it used so much

less frequently than general vocabulary. Repetition breeds memory and repetition takes a concerted effort to learn math vocabulary (Graves, 2009).

In summary, students with SCD are helped by teaching methods that take account of this research into their strengths and weaknesses in their: (a) motor delays make manipulating small items, drawing and writing difficult. (b) speech and language delays lead to their understanding being underestimated. (c) auditory processing and working memory difficulties make learning from listening difficult. (d) strengths in social understanding and enjoyment in learning from social interaction with peers and adults. (e) relative strengths in visual processing and visual memory make learning from seeing important and effective and they are visual learners, and (f) strengths in using gestures to communicate and in showing their understanding by pointing or choosing an answer (Horstmeier, 2004).

Instruction

The most frequently mentioned instructional technique to support the students with SCD is that of systematic instruction (Spooner et al., 2011). *Systematic instruction (SI)* entails these five basic components:

1. Socially meaningful skills;
2. Target skills, observable and measurable;
3. Using data for results;
4. Behavioral principles, reinforcement, prompting and fading, error correction; and
5. Behavior change that can carry over to other contexts, skills and materials (Spooner et al. 2011).

Systematic instruction is, as it suggests, a very specific system of steps that allow for careful planning and record keeping monitoring progress. It defines skills, methods, and

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frequency of practice and assessment and records data emphatically (Spooner, Browder, & Mims, 2011). To plan SI teachers follow four steps: (1) define skills to be acquired, (2) define methods to use, (3) implement the SI plan and frequency, and (4) review student progress to modify instruction with charts, lists, and graphs. SI is the overarching instructional package with strong evidence of effectiveness.

This “evidence-based” practice (Spooner et al., 2011, p. 93) of instruction is encouraged for use, especially for students with SCD. However, in this emerging field of mathematics, evidence-based criteria are more difficult to establish for several reasons. Identifying evidence-based intervention and practice is difficult if there were not enough studies or participants to make conclusions. “Researchers are just beginning to discover what works” (Spooner et al., 2011). In applying evidence-based practices professionals need to be careful to pay attention to prescribed procedures and adapt and evaluate procedures for the individual student. Assessing progress is key. The individual student is the unit of analysis and visual inspection of graphed data is the primary method used to analyze change (Spooner et al., 2011).

Instructional intervention programs should be research-based and proven effective. However, there are intervention programs that do not have the stamp of approval of the “research-based” criteria, yet can provide especially beneficial support. Several research specialists have suggested principles for teaching numeracy and the broader array of mathematic areas to students with SCD. They all corroborate one another and support the overarching, sound bedrock of metacognition, mathematical thinking skills. These features also uphold the premises of SI. Ginsburg, Pappas, and Griffin support 10 insights:

- Guided integration of what children bring to the task, informal knowledge
- Wisdom of culture, what is taught in school, or at least should be

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- General models that immerse children in the world of number
- Draws on their prior knowledge
- Attempts to transform that knowledge into a more mature and formal form
- Align instruction with the natural developmental progression of mathematical thinking
- Provide hands-on games and activities that encourage children to construct meaning
- Encourage communication in spoken language and writing
- Ensure that activities capture children's emotions and imaginations, that is they **enjoy mathematical learning** (emphasis added)
- Create activities appropriate for children from different cultural and social backgrounds (Ginsburg & Pappas, 2007).

Allsopp suggests 13 research-supported strategies to look for in programs and instruction of students with SCD:

- Building meaningful student connections
- Continuous monitoring/charting of student performance
- Dynamic assessment for mathematics
- Explicit teacher modeling
- Instructional games
- Planned discovery activities
- Self-correcting materials
- Scaffolding instruction
- Structured cooperative learning groups
- Structured language experiences
- Structured peer tutoring

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- Teaching concepts/skills within authentic context
- Teach using big math ideas in all five strands (Allsop, 2007).

He also identified four universal features of meaningful mathematics instruction for struggling learners. These features are: (a) understanding and teaching the big ideas in mathematics, (b) understanding learning characteristics of and barriers for struggling learners, (c) making mathematics accessible through responsive teaching, and (d) continuously assessing learning to make informed instructional decisions (Allsop, 2007). Horstmeier (2004) promotes nine general principles for teaching students with SCD:

- Teach in the same stages in learning as other children
- Provide a wide range of progress levels and good teaching helps
- Interactive, social learning situations
- Consider and accommodate speech production difficulties
- Create vocabulary lists that expose and give clear context
- Visual supports that link specific images with the spoken word
- Multi-sensory materials that teach through play and support recall (Horstmeier, 2004).

Three of these strategies stand out as they repeat themselves in all three resources: (a) hands-on instructional and interactive games and activities that help construct meaning, (b) spoken and written language experiences, and (c) developmentally progressive big ideas of mathematical stages and all five strands. Dice, board, and card games, puzzles and teacher created activities such as money, numeral, and bingo (or 'fringo' for the fractions) boards and visual pictures and variety of color of objects will all bring interaction and meaning to mathematics. Dice dots turn into real counting and combining skills for use in sums and products

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in everyday living. 10 frames, Base Ten Blocks, Cuisenaire rods, Fraction Bars and real Hersey bars or other small food items bring number sense and are enjoyable to children. Cutting apart charts, bars, or pictures brings an added dimension to the learning experience. Communicating their steps and expressing what they are doing and why verbally, can bring meaning and vocabulary development (Bley & Thornton, 2001).

Presenting a variety of strategies for concepts helps provide differentiated instruction throughout the development of mathematical ideas and makes students think about different ways of working out problems and they in turn are thinking mathematically, with metacognition. “If teachers teach students how to break problems into small steps, relate symbols to words *throughout the school experience*, and preview vocabulary when needed, students can become successful problem solvers” (Bley & Thornton, 2001, p. 41).

Though these students often lack systematic approaches to identifying and solving problems, there are problem-solving strategies that can be directly taught to students with SCD (Agran, Blanchard, & Wehmeyer, 2000). Many students with Down Syndrome and similar cognitive disability typically reach what Piaget (Piaget, 1950) describes as the concrete operational learning stage. Typical children reach this stage about 7-12 years old:

A child is able to solve problems through mental thought according to logical rules. However, this problem solving can only be done when the child is dealing with **concrete** information that he can perceive directly. If only the abstract numbers are given, he may not be able to set up the problem. These students need hands-on materials to learn math concepts. (Horstmeier, 2004, p. 5)

Students with SCD need to have distinct smaller steps and much more practice before them to organize their thoughts and keep learning in manageable chunks (Horstmeier, 2004) (see Figure 3). Instruction that involves a variety of instructional approaches and a meaningful application is critical. These approaches may be different and more explicit for the SCD children. Some examples include embedding mathematical problems in stories with in the natural schema of life (Browder et al., 2011) (see Figure 4).

Contextualizing math problems provides meaning and bridges the abstract qualities of mathematics that sometimes stump the students with SCD. Music can also make connections for students and be highly motivational (Browder et al., 2006). Calculator instruction and use are uniquely beneficial for the students with SCD to help them get past their calculation or mental math difficulties likely due to their working memory problems (Horstmeier, 2004). Other assistive technology such as computers, iPads, and smart boards are rich sources of support for these students who tend to be more visual learners. Technology sources are usually interactive, colorful, driven by application and engaging (Horstmeier, 2004).

If the expectation is for students to access the many environments of our world through travel, the Internet, the media, and their jobs, then students need the opportunity to comprehend mathematical concepts. The ongoing challenge is to be sure that as mathematics is learned, students also are taught to make connections to their daily lives so that these skills have meaning and utility. (Browder et al., 2006 p. 193)

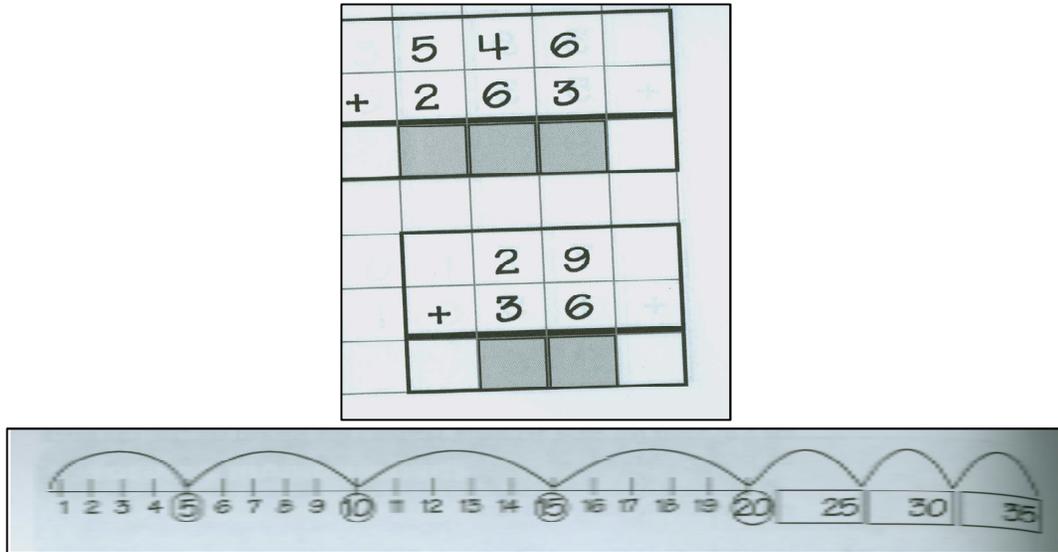


Figure 3. Examples of chunking material into smaller steps and pieces (Horstmeier, 2004).

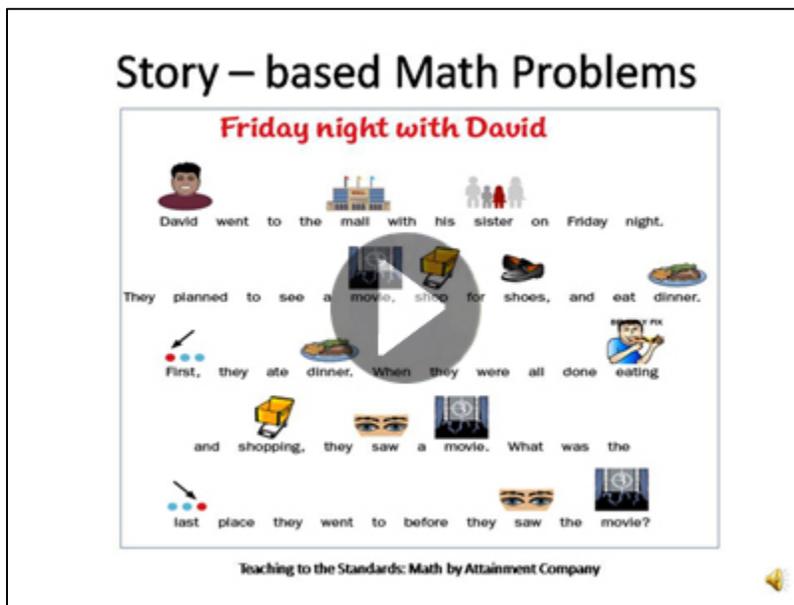


Figure 4. Story-based math problem examples (http://mast.ecu.edu/modules/sscd_mc/concept/)

Interventions

Mathematic materials for instruction come in many different forms. Complete curricula of each school are important to consider and support. Several examples of common curricula for the general mathematic classes presently are Investigations, Everyday Math, MATH

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Connections, Teaching to Standards: MATH, and Bridges. “When planning the use of alternative materials, it is important to use the general education class materials whenever possible” (Browder et al., 2006 p. 188). Many computer- assisted programs are available for a wide spectrum of support, particularly memory training and capacity building.

Information below distinctly compares Intervention programs (see Table 1). In special education classrooms it is generally an intervention program that forms the basis to the instruction for students with SCD (Griffin, 2007). “Until recently, these programs were produced almost exclusively by commercial publishing companies” (p. 374). These past 20 years have brought education and cognitive science psychologist experts in to help in this arena to try to develop intervention programs that would be more effective. The following is a comparison of the most commonly mentioned in all of my research and the attributes of such that support my findings of better instructional practices. The seven most mentioned intervention programs in my research were the following.

Do the Math was created by renowned American Marilyn Burns. It is a number- based intervention program that focuses on fluency with whole numbers and fluency with fractions. The program scaffolds these topics and addresses these research- based practices: Explicit Instruction, Multiple Strategies, Gradual Release routines, Student Interaction, Meaningful Practice, Assessment & Differentiation, and Vocabulary and Language (<http://teacher.scholastic.com/products/dothemath/po.htm>).

Voyager or VMath provides instruction that balances conceptual development, computational fluency, and problem solving. It covers all the strands of mathematics and provides hands-on lessons to help teachers present important math concepts using common manipulatives (<http://www.voyagerlearning.com/vmath/curriculum.jsp>).

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ADD Vantage/Math Recovery touts the phrase Learning Framework In Number (LFIN), and provides participants with an understanding of how children learn mathematics. It helps teachers assess their students accurately and custom-fit their instruction to enable students to progress in the area of mathematics (<http://www.mathrecovery.org/>).

Number Worlds builds foundational skills and concepts, and makes learning fun. It builds foundational math skills and prepares younger children to understand more complex concepts later. It looks to be the most comprehensive program that helps children structure and understand the world.

Kumon and Numicon are both United Kingdom programs that are often used supportively. Kumon is strictly numeracy drill and practice that improves automaticity and has been found to be very effective for this and research supports growth in computation. Numicon then provides more unique multi-sensory manipulatives especially for counting, ordering, and subitizing and provides a combination of action, imagery and conversation. Both are numeracy-based programs (<http://www.kumon.co.uk/>) (<http://www.numicon.com/Index.aspx>).

Math Perspectives is the newest intervention program and it has the most variety of excellent numeracy materials and an exceptional computerized assessment system making it very efficient and easier for teachers. Exceptional variation and innovative analysis are its strength.

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Table 2

Comparison of Intervention Programs for mathematics

Intervention	Evidence/Research Based	5 Strands	Primarily Numeracy	Systematic Instruction	Hands-On/ Concrete
Do the Math	X		X	X	X
Voyager	X	X		X	
ADD Vantage/Math Recovery			X	X	X
Number Worlds	X	X		X	X
Kumon			X	X	
Math Perspectives	X		X	X	X
Numicon	X		X	X	X

Note. This chart compares the components of Seven Mathematic Intervention Programs

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The intervention program cannot be the only means of instruction or drive the teaching of the students with MLD or SCD. The complete repertoire of strategies previously mentioned should be considered and implemented according to individual needs. All of these interventions programs promote strong number sense, but more intervention programs need to be created that support all the strands and provide systematic instructional and research-base to all the strands. It is clearly hard work (Bicard, Bicard, Casey, & Nichols, 2008).

Functional Mathematics

Although the focus of this review is on academic mathematics, it is safe to say that most students with SCD are provided instruction in functional mathematics, too. Thus, it is appropriate that this area is addressed to some extent. Functional mathematics involves skills that prepare students for life such as grocery shopping, preparing a meal, cleaning, negotiating transportation, and paying bills. Preparing for jobs and work are an ultimate goal (Browder and Spooner, 2006). Money skills such as purchasing, comparing, computing, and budgeting will develop all the numeracy work. The “Next Dollar Strategy” involves excellent number sense and focuses on estimating to the nearest next or lower dollar rather than dealing with confusing coins and making change. Calculator instruction and practice are essential here (Browder & Spooner, 2011). Many students with SCD are meticulous about order so record keeping, charts, planning, and banking skills can be taught well. Time management, schedules, and pictures are all still a good place for mathematical instruction and teaching meaning. Mathematics embeds easily into these skills and it can continue to do so as the standards based learning is also advanced and broadened. It might be helpful if there were no distinctions between general and functional mathematics goals. However, daily living skills may not be taught well in the general curriculum and should be specified in IEP goals (Horstmeier, 2004).

Chapter 3

Discussion

Implications

Three critical limitations seem distinctive throughout my research. First, there is a limited amount of research available for understanding and implementing quality educational strategies and curricula for students with severe cognitive disabilities. Two decades of research are proportionally lacking compared to over a half century of reading research. “Mathematics has always been an afterthought in the learning disabilities field” (Gersten & Chard, 1999, p. 25). There are few to no studies that focus on longitudinal development in more complex numeracy, data, and algebraic principles (Kleinert et al., 2009). We “envision” (Gersten & Chard, 1999) research that parallels that of reading and effectively develops and assesses number sense, mathematical reasoning, and problem solving even for the students with SCD. Related research in the fields of development, neuroscience, genetics, and diagnostic instruments needs to be conducted.

Second, special education itself is also a product of the second half of the 20th century. National and state emphasis on education, groups such as NCTM and NCSM, and shortages in qualified educators have impacted the education community. New general curricula are developing that offer broader opportunities in the five strands of mathematics and problem - based learning. Classroom teachers and special educators need training, resources, experience, and practice (Kleinert et al., 2009). “Limited research on academic content instruction for this population creates a challenge for practitioners seeking examples from the literature” (Browder, 2003, p. 161). It is critical that intervention programs continue to be developed. Future IEPs for

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students with SCD “will undoubtedly contain a blend of academic and functional objectives” (Browder, 2003, p. 161). They will benefit from whole group and individual instruction.

Third, special educators are presented with the enormous task of being knowledgeable about so many areas of student disability. They must become experts in all the diagnosed disabilities such as learning disabilities, severe physical *and* cognitive disabilities, emotional disabilities, and the ever-increasing disorders such as oppositional defiance, attention deficit disorders, obsessive compulsive, among others. NCLB is beneficial for the students with SCD but their teachers have been stretched beyond measure (Bicard et al., 2008). These special educators are then also often put in charge of three to 10 adults to help support special education students. They are then put in the position of training adults in an already taxing job with incredible variance of need. Many special education training degrees provide only the tip of the iceberg in learning best instructional practices, especially in academic areas. We teachers learn best through experience. Sadly, special education teachers are limited and often changing positions or leaving altogether (exit attrition), and this makes it difficult to become experts (McLeskey et al., 2004).

Why can't general education teachers fill some of these gaps in the great shortage of special educators? General education teachers could do the job of many of the para-educators that the system uses to help these special education classrooms and children in them. General education teachers could share their strength in academics with the special educator's expertise in disability, and there could be much accomplished if we worked with one another instead of the conflict and separation often seen. Most recent data indicates that about 86% of teachers were prepared for each available position in special education, while more than twice as many teachers were produced for each available elementary position (Bicard et al., 2008). We need to

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come up with strategies that address this immediate shortage with research, recruitment, retention, diversity, and policy (McLeskey et al., 2004).

It is intriguing that much research in mathematics has been conducted in Europe. Hopefully, specialists and researchers will be communicating more around the world with the onset of technological sources to share information and findings. All will benefit. This field of mathematical instruction for students with MLD and SCD should be increasing in the coming years. If the national mandate to include these children in general curricula is receded, will there be enough highly qualified teacher and knowledgeable professionals? Hopefully, teachers at all levels and across all areas of expertise will become more aware of the potential academic *abilities* of students with SCD and act accordingly.

Conclusion

Dr. Elizabetta Martinez (1999), mathematician and researcher at University at Padua in Italy, has documented essential conclusions of over 20 years of research on progress of students with Down Syndrome. She and other specialists suggested that these students progress through the same stages of development as typical children. She argues that the ability to do arithmetic is not essential to understanding other areas of math such as geometry, problem solving, data, algebra, and measurement. “They follow the same path as their typical classmates at a slower rate, with more steps and with individual teaching” (Bird & Buckley, 2001 p. 17). Martinez argues that the ability to learn arithmetic facts and mental math is not essential to understanding other areas of math such as geometry, problem solving, data, algebra, and measurement. Italian schools have been fully including their students with disabilities for over 20 years. The United Kingdom (UK), the Netherlands, Finland, and Australia also have done much research in this area. If teachers, parents, and case managers could cooperate in meaningful mathematic IEP

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goals for students with SCD, the discrepancy between content standards and functional skills could be minimized and a full and dynamic mathematic instruction could become the new normal. As Martinez (1999) notes:

Students with learning disabilities can succeed in academic programs, where even typical students may have difficulties, and can enjoy studying these programs. If we believe the academic culture is precious and pleasing for us, why should we not share it with people with difficulties? If it helps us, why should it not help them? I think the right path might be a fair balance between academic programs and training for autonomy” (p. 16).

Summary

Mathematics education is changing in many ways. New, dynamic, and balanced standards have weathered the storm of ‘old math’ and ‘new math’. Conceptual and procedural knowledge are interactive and influence and trigger one another (Gersten & Chard, 1999). These standards will hopefully bring a strategic, broadened, and meaningful knowledge to mathematics teachers and students. Cooperation instead of competition should reign. Better understanding of cognitive disabilities should provide critical strategies in instruction and allow the recently acknowledged potential of students with SCD to expand and flourish. Technology and communication capabilities are providing exceptional resources to be created and implemented among the mathematic and special education sectors of education. Mathematics instruction will be enhanced if the community of teachers will continue to be learners and become more knowledgeable and master this teaching.

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