# EXPLORING STUDENTS SUMMATIVE UNDERSTANDING USING SCAFFOLDED TASKS IN TWO ALGEBRA 2 CLASSES

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# Abstract


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Often in mathematics students are asked to reproduce information. Students are frequently expected to  regurgitate algorithms and definitions rather than apply mathematical concepts and show their understanding of the material(Iannone & Simpson, 2015, p. 979). The reproduction of information creates a transactional process in where the student is seen as the processor and the instructor is the “knower” of knowledge and there is no mastery demonstrated from the student(McFeetors et al., 2021, p. 1). Students do not always have the opportunity to show their incomplete understandings and mastered steps to solving larger problems in traditional classroom practices.

Scaffolding is an instructional method used to facilitate students by providing support for them to build on their prior knowledge to then discover new information (Ahmed, 2010). Scaffolding within tasks can create the opportunity for students to show mathematical mastery by using previous knowledge to build connections between mathematical concepts. The connections the students create between scaffolded questions in an assessment can help students show their mastery they would not be able to independently as they are given the chance to recall prior knowledge. In my research I explored whether scaffolds aid students in showing mastery of a given mathematical standard when compared to the shown work and skills used of students who are not given scaffolds. The scaffolds implemented in the assessments are based on the Wyoming State Mathematical Standards performance level descriptors. It is my intention that the information on the use of scaffolded assessments could better prepare my students in mathematical mastery rather than reproduction of information.

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# Chapter 1

## Introduction

As you walk into the classroom, you are shocked when you realize there is an exam. With lack of preparation and a heightened amount of anxiety, your best attempt is made. During the next class period you brace yourself to see all the red markings left by your teacher. "Assessment has been viewed as transactional, where students give while teachers take and then teachers 'pay' through giving marks while students take on the labels the marks dictate" (McFeetors et al., 2021, p. 1).

The transactional process can create a disconnect between the communication between students and teachers. The disconnect creates a lack of learning as the transaction and markings do not accurately show the understanding of the student. Assessments are at the heart of this disconnect as they are the summative means of communication between teachers and students. It is challenging for an educator to look at a summative assessment that includes a mixture of multiple choice and short answer questions, especially when they are trying to get a clear view of the connections a student has made between the different mathematical skills learned.

Assessments should be created as a means of communication within the classroom where the student can explain their understanding and the teacher can provide an opportunity for feedback (Liljedahl, 2010, p. 7). The transactional process provides less understanding of students' knowledge, as there is not much the student has to do to complete the standardized, summative assessment. Summative assessments should be a place for students to connect and apply their mathematical understanding but is often used as just a dumping ground for regurgitated information.

Students often see summative assessments as a place to unload the information they have learned using memorization rather than applying their knowledge (Iannone & Simpson, 2015, p. 980). As stated by Martínez-Sierra et al, "[Teaching mathematics is] helping students to develop their ability to reason, helping them to understand its importance; that they know that, someday a formula may not help them, but reasoning will"(2020, p. 494). Assessments should aid in creating these opportunities for students to apply their knowledge. For example, providing the opportunity for students to connect the material they have learned through an assessment allows for students to be engaged and find purpose in their learning.

## Statement of Problem

On the whole, students are not able to accurately show their understanding by creatively solving problems because they do not always have the necessary recall and often do not relate to the material. More specifically, their inability to relate to the material results in a surface level understanding of skills learn in class. Further, unless the exact problem has been modelled beforehand, students often fail to demonstrate the skills being assessed. Current assessment methods in the mathematics classroom call for change, as assessments have been seen to "encourage students to memorize facts rather than think and solve problems creatively." (Shahbari & Abu-Alhija, 2018, p. 1317)**.** As students begin to venture outside of the classroom, it is imperative that they learn to creatively problem solve because such skills are necessary for the real world.

For mathematics to be useful, students need to understand how to connect the skills being taught in the classroom with problems that occur outside of the classroom. A change to summative assessments can provide students the opportunity to see these connections with the help from their educators. Students need to be given the opportunity to show their understanding through the support of better created summative assessments.

To aid in the creation of connections, scaffolded assessments with real world application could be created by educators to put more learning and application of material on the students. Scaffolded assessments can supply students with the necessary support to create these connections as a summation of knowledge as " in subjects like mathematics, when failing to answer the first part of an examination question correctly often results in the inability to answer the subsequent parts." (Iannone & Simpson, 2015, p. 973). These scaffolds can include problems throughout an assessment where guidance, given in the directions of the assessment problems, on how to solve dissipate as students continue to work through the assessment.

As scaffolding creates the support for students to complete a summative assessment, the problems being asked within the assessment are equally as important. To encourage the change of students showing their learning, several research studies that suggest the use of authentic tasks be used within summative assessments as "Authentic tasks align to 21st- century skills supporting student centered approaches with problem- based learning and project- based learning" (Stobaugh, 2013, p. 67). Assessments need to give students the opportunity to interact with mathematics in a manner that can be implemented outside of the classroom. Using authentic tasks within assessments allows students to form useful critical thinking skills and communication skills that can be used in the real world. Therefore, giving students opportunities to interact with authentic tasks, so they can show their summative understanding, allows for connections with the outside world to occur.

## Purpose

In this project I plan to explore the ideas of scaffolded assessments that include authentic tasks. This is in part because I have also noted, like McFeetors and colleagues (2021), the stress that comes along with assessments is often correlated with anxiety, because they are often heavily weighted on students' grades (pg.8). Although the weight assessments carry may not decrease, the engagement and preparation for students can lessen the burden.

I have found that it is often difficult for my students to organize and understand the connections between the different mathematical skills on an exam due to added stress. Iannone and Simpson (2015) found something similar in their work. As students in their study were working through an exam they noted, "if you don't know a question, you can get stuck and you can't go any further with the question but if you get a little bit of guidance maybe you can show that you can work through a bit more. You've got a second chance" (Iannone & Simpson, 2015, p. 982). Several mathematical studies suggest that scaffolding can assist students in completing tasks, but there is very little research on the effects of scaffolds in mathematical assessments. Scaffolding within an assessment can create the type of guidance students need to tie ideas together and complete an assessment that includes their understanding of several different skills (Abraham & Jones, 2016, p. 39).

Scaffolds can assist students in continuing to solve problems, but it is also necessary for students to also be engaged with the work. Stobaugh suggests that " real examples from the students ' local community and those that relate to their interests can pique student attention in the topic studied." (p. 66). Including authentic tasks within assessment can aid in promoting engagement that encourages students to create connections using their mathematical understanding. The resulting connections can then result in a longer ranging learning opportunity. As Stein states, “Application rather than retention becomes the mark of a successful instructional encounter” (p. 3).

## Research Questions:

This research project is guided by the following questions:

- **How do scaffolded summative assessments help show students mathematical understanding in Algebra 2?**
  - **Do scaffolds help students connect mathematical ideas?**
  - **Are students able to draw more mathematical connections when completing authentic tasks?**

I plan on diving into finding new methods to assess students by further evaluating their understanding. Creating scaffolded, summative assessments gives students the support they need to connect different mathematical skills in an engaging manner, using authentic tasks. Scaffolding an authentic task allows students to organize their knowledge in a supportive manner, so that they can more clearly show their understanding of the summative material.

## Chapter 2

## Literature Review

### Theoretical Lens

The *Zone of Proximal Development* (ZPD) is a term often used in U.S. Education. Vygotsky introduced the concept in the late 1920's to criticize the testing in Russian schools that reflected only the current level of learners' achievement and did not include potential for student development in the future (Shabani et al., 2010, p. 3). ZPD is defined as a gap between what students can perform individually and a task, they may need teacher guidance to complete, creating the potential for development (Ahmed, 2010, p. 2). Shabani et al. (2010) states "The idea is that after completing the task jointly, the learner will likely be able to complete the same task individually next time, and through that process, the learner's ZPD for that particular task will have been raised." (p. 2). The Zone of Proximal Development encourages the opportunity of students to create connections and further their mathematical development when implemented in the classroom.

The Zone of Proximal Development can be implemented in the mathematical classroom for better connections to be created between skills students already know and something they do not know (Ahmed, 2010, p. 2). Ahmed (2010) found evidence supporting ZPD as students taught in a classroom that included attention to addressing the ZPD of learners had greater mathematical achievement when compared to students taught in a more conventional classroom. This evidence leads the idea that scaffolds within assessments can be a change needed in the mathematics classroom that could create connections between concepts and a better shown understanding of students' mathematical knowledge.

Scaffolding can be created in assessments using a ZPD lens. These scaffolds can aid in students bridging former knowledge to apply to new information to further their mastery of mathematical concepts.

Similar to Vygotsky's ZPD , Situated Learning is a theory that emphasizes connections made by learners while engaged in learning .  The learning explicitly reflects and draws implications from previous experiences through the immersion of new experiences (Stein, n.d., p. 3). New experiences can be created using authentic tasks. Authentic tasks used in situated learning can further create connections for students in which they are applying classroom knowledge with previous experiences.

Situated learning connections include higher order thinking that relates to the real lives of students (Stein, p. 3). Through the use of scaffolds, students can engage in realistic and problem-centered tasks to further their connections with mathematical concepts (Stein, p. 5).  The connections created with these tasks aid in students creating meaning with the learning (Stein, p. 2). As Stein states, "Situations are presented that challenge the intellectual and psychomotor skills learners will apply at home, in the community, or the workplace" (p. 2). Situated learning with the use of scaffolds allows students to gain problem solving skills they can apply not only to assessments, but also to their lives outside of the classroom.  Using Situated Learning Theory, "Application rather than retention becomes the mark of a successful instructional encounter,"  as connections between the mastered skill and the outside world are created (Stein, p.3).

## The Problem with Summative Assessments

An issue with mathematical assessments is that they often ask students to reproduce information, where they are expected to apply what they have been shown in the classroom rather than showing their understanding of the material (Iannone & Simpson, 2015, p. 979). Summative assessments in mathematics creates a transactional process, wherein the student is seen as a regurgitator of information and the instructor is the “knower” of knowledge and there is no demonstration of mastery from the student (McFeetors et al., 2021, p. 1). In many classrooms, students are not given the opportunity to show their understanding and mastery using a traditional summative assessment. Students do not need conceptual understanding in traditional assessments as they often just need to memorize procedures and apply them (Shahbari and Abu-Alhija, 2018, p. 1327). This lack of understanding in assessments calls for a change.

The lack of demonstrated understanding from students on traditional tests is cause for concern, as assessments are often used to guide instructional decisions while providing students with feedback regarding confusions and potential gaps (Shahbari and Abu-Alhija, 2018, p. 1316). Mathematics is a foundational subject that builds on itself, and it is imperative that student gaps are addressed.  In example, Ahahbari & Abu- Alhija explain the problem thusly: “It should not, therefore, be limited to achievement tests that summarize performance at the conclusion of the instruction and thus encourage students to memorize facts rather than to think actively and solve problems creatively” (2018, p. 1317).

The weight associated with summative assessment can create a sense of anxiety for students, hindering them from showing their true mathematical understanding (McFeetors et al., 2021, p. 8). Shahbari & Abu-Alhija  explain that “Alternative assessment can provide students with an opportunity to demonstrate their true ability. ( 2018, p. 1327).   Similarly, in the study conducted by Shahbari & Abu-Alhija, a teacher recalled getting test anxiety and forgetting test material or the method needed to solve the problem (2018, p.1327).  On the whole , educators and students can relate to the weight of summative assessments, which shows a need for a change in the way mathematics students are tested for their understanding.

## Scaffolding for a Solution

Scaffolding can be defined as a method used to facilitate students building on their prior knowledge to incorporate new information (Ahmed, 2010, p. 2). Scaffolding within assessments can also create opportunities for students to show their understanding using previous knowledge to build connections between mathematical concepts. Iannone & Simpson summarize the functionality of scaffolding in the classroom when they write, " In an exam, if you don't know a question, you can get stuck and you can't go any further with the question- but if you get a little bit of guidance maybe you can show that you can work through a bit more. You've got a second chance."(2015, p. 982). The connections the students create between scaffolded questions can help students show their mastery of a summative unit when they would not be able to do so independently (Ahmed, 2010, p. 2).. Using ZPD to scaffold in assessments, students are given a chance to recall prior knowledge.

Moreover, scaffolded assessments can create opportunities for teachers to have a better understanding of their students' summative knowledge. A student in a study that used the Vygotsky approach with a scaffolded assessment stated:

> "It gave me a better understanding of the basics of financial analysis that was covered throughout the subject. It allowed me to incorporate what I learnt into an assignment and that was really beneficial in understanding key information"(Abraham & Jones, 2016, p. 38).

Ultimately, the scaffolds within assessments allow all learners of various levels of expertise the opportunity to incorporate their summative knowledge (Herrington & Oliver, 2000, p. 26).

## Applying Mathematics to the Real World

Authentic tasks within an assessment can be a method used for students to continue building connections between mathematical concepts and application to the real world. Authentic tasks can be defined as an event that could occur in real life, a question posed in that given event, or a task that includes real life data (Vos, 2018, p. 5). Students can use authentic tasks to help them make connections between what they are learning inside the classroom and situations they may be exposed to outside the classroom.

Preparing students for the world outside of the classroom is essential. In support of this theory, Stobaugh states that "Authentic tasks align to 21st- century skills supporting student centered approaches with problem- based learning and project- based learning" (2013, p. 67). To show mastery of a given standard or skill, students should be able to use their understanding of mathematical skills to think actively and solve problems creatively (Shahbari & Abu-Alhija, 2018, p. 1317). Students' applied understanding of mathematical competences to creatively solve authentic tasks creates skills that can then be applied outside of the classroom.

The creative solving students must use to complete an authentic task provides access to the highest order of thinking according to Bloom's taxonomy, at the create level (Stobaugh, 2013, p. 68). The create level of Bloom's taxonomy includes students investigating and formulating ideas in order to solve unique problems relevant to their everyday lives; similarly, "authentic tasks simulate job challenges, requiring research and multiple steps to create the solution or product (Stobaugh, 2013, p. 67). The higher level of thinking students must complete to solve authentic tasks further aids in the development of 21-st century skills, which are becoming essential outside of the classroom.

Motivation to understand mathematics in particular comprises a unique challenge for students, as they often do not see the connection between their personal lives and what is being learned in the classroom. Authentic tasks allow students to incorporate skills learned in the classroom through assessment, thereby aiding in the understanding of key information (Abraham & Jones, 2016, p. 38). Highlighting classroom skills through students' completion of authentic tasks provides opportunities for students to show true mastery of a given skill as they are applying information through created connections rather than merely retaining the information for a given exam.

The implementation of the Zone of Proximal Development in daily classroom practices through the use of scaffolds can provide students the opportunity to connect classroom skills on an assessment. Using scaffolds within daily classroom practices can facilitate in students building on prior knowledge to then internalize new information (Ahmed, 2010, p. 2). The scaffolded build between prior knowledge and the internalization of information within daily classroom practices can aid in creating stronger connections for students to further show mathematical mastery.

By utilizing the ideas brought on by Vygotsky's Zone of Proximal Development and the Situated Learning Theory, a change can be made in the formulation of summative mathematical assessments. Using the scaffolds discussed in the ZPD, students can show their knowledge when completing authentic tasks. Scaffolding creates opportunities for students to connect prior information to tasks that may be found in the real world to show their true mathematical knowledge. In conclusion, applying scaffolds and authentic tasks to mathematical summative assessments could aid in students' mastery of skills rather than short term retention.

## Chapter 3

## Methodology

I conducted research to explore if scaffolded summative assessments could aid students in showing their mathematical understanding in my Algebra 2 courses. I created a scaffolded and unscaffolded assessment to contrast against one another using the Wyoming Mathematics Performance Level descriptors. The scaffolded assessment included questions based on the basic, proficient, and advanced level descriptors. The assessments (see Attachment A) included questions 17-18 at the basic level, questions 19-21 at the proficient level, and question 22 representing the advanced level performance descriptor. These three levels provided scaffolding for mathematical thinking as students are asked to define, perform and apply the mathematical skill for the given standard as they progress through the different performance levels. The unscaffolded assessment (see Attachment B) assessed the students understanding of the mathematical standard at hand without any scaffolds within the individual questions. The goal of my research was to see if students could show more understanding of the connections created between mathematical concepts and apply them using scaffolding.

After noticing that my students struggle to connect different mathematical ideas together and relate them to their lives. The assessments I have created use information from prior units to further connect ideas by posing a question that could be asked in a real- world event, which can be defined as an authentic task (Vos, 2018, p. 5). The final question on each assessment was an authentic task. The authentic task had students apply their understanding of the mathematical standard to aid in students connecting their problem-solving skills to a real- world event. After reviewing the literature, I was motivated by the idea that students can show mastery of a skill when they are given the proper supports (Abraham & Jones, 2016, p. 30). Scaffolds can support students in engaging tasks that are relatable to skills or problems outside of the classroom.

To analyze the use of scaffolds, I evaluated the work of students on the assessments with and without scaffolds. It was my goal of the evaluation to analyze the themes that emerged from the students' work on the two different assessments. It was my goal to see if the work of students who were given an assessment with scaffolds would show more understanding of the assessed standard and appropriately use the learned skills as they progressed through the different performance level descriptors. Qualitative data was collected to set a baseline for each student's basic understanding of the standard being assessed. Qualitative data was also used to determine if there are any emergent themes that appear between the different assessments.

I also analyzed data based on the shown mathematical work of students on the short response problems on the assessment using a case study approach. There were five assessments analyzed for emergent themes. These case studies were individually evaluated and then compared across cases to assess different themes. The individual cases were assessed to see if the scaffolds had an effect on aiding students to a correct solution. The cross- case analysis looked at if the scaffolds created a difference in the solving process or solution.

**Positionality**

I was interested in conducting research regarding summative assessments in the classroom because I have felt a large disconnect within what my students can apply on an assessment. As I would grade assessments, I often found myself questioning how students are solving and what skills they would be applying as their answers often did not make sense within context of the given problem. When lecturing, I complete an example for the students, have the students complete an example with their peers, and then assign an individual assignment for them to later apply the specific skill to. The gradual release of students individually applying the skill mimics the type of scaffolding I wanted to see if students could apply on their summative assessments. Scaffolding within the assessment allowed me to still give structured instructions to aid students in seeing the connections in each problem and then eventually giving students the opportunity to apply their understanding of the given concept or skill.

While grading assessments I also find students struggling to make sense of a problem because the context does not relate to their lives outside of the classroom. I created summative assessments that included authentic tasks to help students bridge between the mathematics classroom and the outside world. It is my hope that students can use this bridge to show a clearer view of their understanding of a specific concept. Using authentic tasks could aid in students using the scaffolds to show mastery of a concept by being able to connect to the context of a mathematical problem.

## Population

This study was conducted in two Algebra 2 courses in a Wyoming high school. The demographics of the high school where the study was conducted are shown in figure 3.1 and 3.2 from https://www.publicschoolreview.com/central-high-school-profile/82009.

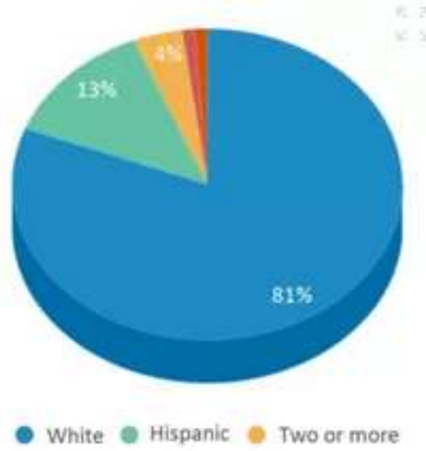


**Figure 3.1:** Ethnic Groups

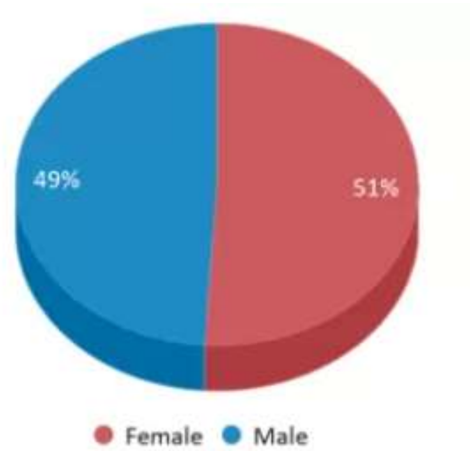


**Figure 3.2:** Gender Percentage

The Algebra 2 courses include thirty-four total number of students. There were 15 students in class A and 19 students in class B. Class B is a cotaught class with 4 students being on an IEP or 504 plan. I chose to study an Algebra 2 course as they were easy to apply a change in instructional tasks with it being my own class and there were two of them, making a comparison between assessments easier. The concepts covered in Algebra 2 also included standards that were easy to apply the different Wyoming State Performance level descriptors to an authentic task in hopes it would further encourage students to show their mathematical mastery.

**Participants**

All students in the Algebra 2 Class A completed the scaffolded assessment. From this class there were two students who provided consent for their work to be used. Students in Class B completed the non- scaffolded assessment in which three students provided consent for their work to be analyzed. Students provided assent and their parents provided consent using Microsoft Forms. The forms were then sent to the districts mathematics coordinator, who matched the consent and assent of parents and students. Once students provided permission for their work to be used, their assessment was anonymized and sent back to be analyzed.

## Data Gathering and Materials

A scaffolded assessment will be created using the Wyoming Mathematical Performance Level descriptors. The scaffolded assessment was based on the standard A.APR.D.3: Identify zeros of a polynomial function when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. It will include questions at the basic, proficient, and advanced level based on the Wyoming state mathematical performance descriptors. The advanced level will consist of an authentic task in which the skills used in the previous levels of questions must be applied to a problem that is authentic to a real-world scenario.

The scaffolded assessment includes questions from the basic, proficient, and advanced level where each question builds on each other and has scaffolds built into each level to support the students in showing their understanding. At the basic level of the given standard, the "Student is able to identify zeros of polynomials when suitable factorizations are available, and locate the zeros on a coordinate plane" (*2018 Wyoming Math Content Standards & 2020 Performance Level Descriptors*, n.d., p. 111). Students were assessed on two questions, #17 and #18, at the basic level in which they must identify the zeros of a polynomial function given an equation and given a graph. Building from that question, students move onto questions #19 through 21, at the proficient level, in which "Students is able to identify zeros of a polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial" (*2018 Wyoming Math Content Standards & 2020 Performance Level Descriptors*, n.d., p. 111). At the proficient level, students thinking is scaffolded through the different steps in order to identify the zeros of a function and construct a rough draft of the given function. The advance level was assessed at on question #22, which states that "Student is able to, given a graph of a polynomial function with integer x- intercepts, write the general form of the polynomial in standard form"(*2018 Wyoming Math Content Standards & 2020 Performance Level Descriptors*, n.d., p. 111). At this level students are assessed by being scaffolded to find the standard form of a polynomial given a graph and then applying that information in a real-world context.

The unscaffolded assessment includes the basic, proficient, and advanced levels as well but does not include the aid of scaffolds throughout the given question. The questions ask the students to show their understanding of the mathematical standard based on the different level descriptors. For example, at the proficient level students must show the zeros of a polynomial function and sketch a rough graph. The non- scaffolded assessment asks students to identify the zeros and sketch a graph where the scaffolded assessment provides students with the different steps to find the zeros and sketch the graph. The students work on the same questions on each assessment will be compared qualitatively to see if patterns emerge.

## Analysis

The data was analyzed qualitatively using emergent themes to see if any patterns appear from students' work on either assessment. Each assessment included the same questions based on the basic, proficient, and advanced performance level descriptors stated by the Wyoming State Standards. The assessments differed in the amount of scaffolding within each question. The non-scaffolded assessment only included questions that are explicit to the level descriptors and offered no aid for students to complete the question. The work shown on questions that are the same on the scaffolded and unscaffolded assessments were analyzed for reappearing themes. The themes were then compared to see if there was any evidence that students performed better on either assessment.

The basic level questions were assessed on both assessments as a baseline to see if students could show mastery of the given standard as explicitly directed. The basic level could lead to further understanding of the type of misunderstanding students have based on their completion of the specific skills. The holes in understanding could then be compared to any emerging themes found within each assessment.

The work of students was explicitly evaluated using a case study approach. A case study approach was taken to "illuminate a decision or set of decisions: why they were taken, how they were implemented, and with what result" (Yin, n.d., p. 174). I used a logic model to look at any cause and effect patterns that arose throughout the work of students given scaffolds within an assessment (Yin, n.d., p. 186). A logic model case study allowed for a qualitative analysis to be used to compare consistency between the work of students. By using a case study approach, I planned to be able to identify if there are any patterns that appear within each case by looking at the different variables within the assessments.

# Chapter 4

## Analyzing the Data

The work of five students on a scaffolded or non- scaffolded assessment was analyzed to determine if scaffolds within assessments could aid in students creating connections and show further mastery of a given mathematical standard. Of these five assessments, three were non-scaffolded and two were scaffolded. The mathematical solving process on the questions aligning with the basic, proficient, and advanced performance level descriptors for standard A.APR.D.3: Identify zeros of a polynomial function when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial, were examined. The mathematical processes were then analyzed to determine if there were any reappearing patterns within the individual cases as well as across cases.

### Similarity between Cases

Several students were able to show mastery at the basic level, which were the first questions analyzed, by stating the zeros of a polynomial function given an equation or graph. These first two questions showed similar patterns as almost every student was able to state the correct solution, but lacked work or an explanation of their solution. The basic level was used as a baseline to assess students' knowledge of the given standard as students would need to show that they understood what a zero of a polynomial function was to be successful through progressing questions. Four of the students were able to correctly identify the zeros of a polynomial function given an equation and three of the students were able to correctly identify the x- intercepts of a given graph as the zeros. There were only slight mathematical errors and no attempt made by the students who did not correctly identify the zeros as the basic level.

Throughout the cases there were a few emerging themes within the assessment, regardless of if the assessment included scaffolds. A common theme that emerged was directions not being clear in problem 19 as students were asked to divide a polynomial given a factor and then use that division to aid in finding the other zeros of the polynomial. Students often used their solution, or quotient, from dividing the polynomial to answer the sequential parts when the original polynomial should have been used to answer the remaining parts. The unclear directions may have caused several students to make the same error of using the wrong expression to answer the remaining parts of the given question.

Another theme that was very noticeable was the process of students using synthetic division to solve a variety of different questions within the assessment. In all of the assessments analyzed but one included students using synthetic division to solve for the zeros of a given polynomial. Students were taught several methods to find the zeros of a polynomial and were only directed to divide the polynomial once, but still often would use synthetic division to find remaining zeros. Of the four students who synthetically divided, there were very few errors and all, but one student obtained the correct solution. The commonality of synthetic division highlighted this skill as a strength among several students.

At the advanced level, all students were able to identify a factor from the zeros of the given graph but were not able to connect their solution to the progressing parts of the question. Students used the x- intercepts shown on the graph to create factors and all but one student was able to correctly write the factors given two of the x -intercepts. There seemed to be a lack of understanding regarding the vocabulary in the directions for several students as they identified the last x- intercept as fraction and wrote the factor including a fraction instead of an integer as stated in the directions. Although there was a pattern of identifying similar factors, there was also a pattern in not understanding how to apply those factors to solve the problem. There were two students who wrote solutions without explanations and two other students who correctly started to solve but were not able to complete the problem to find the correct solution. Comparatively, several of the students' work showed similar mistakes and similar understanding of how to solve for the zeros of a polynomial given an authentic task at the advanced level. The emerging themes throughout the different assessments highlighted the mathematical strengths and weaknesses of the students.

## Building Bridges

Throughout each case, there was an emerging theme of students building on skills from previous problems. Each assessment was examined at the basic, proficient, and advanced level. As students progressed to the advanced level it could be seen that they often used skills from the previous performance levels. Regardless of the type of assessment given, it was shown that building questions based on the Wyoming performance level descriptors provided the most aid in students showing their mastery of the given standard.

Case 5 was a non- scaffolded assessment where the student continuously applied similar methods as they progressed through the exam. To build the problems in complexity, as students progressed through the non- scaffolded exam there were less scaffolds given within each problem to identify if students could continue to show their mastery of the given standard. In case 5, the student used methods explicitly asked in prior problems to solve other questions with less guidance. For example, in problem 19 students are asked to divide the polynomial to aid in finding the zeros of that given polynomial. Student 5 used the method of division by a zero directed in problem 19 to find the zeros in problem 21 where students were simply asked to find all zeros given a polynomial. Although the student had minor errors that halted their process in finding a solution, it appeared that were able to build on prior skills throughout the assessment.

Case 1 was very similar in they often used skills in prior questions to complete the progressively more challenging problems. Case 1 was a non- scaffolded assessment in which the student understood the importance of finding factors of a polynomial to find the solutions. The student correctly identified the zeros from factored form at the basic level and continued to attempt to factor each polynomial to find the zeros. After evaluating the work of the student it could be seen that the student created bridges between the problems and often used skills from prior problems to attempt the progressing questions.

There were also two more cases that built upon the solving methods presented at the basic level to solve at the proficient and advanced level questions.  In cases 3 and 4, they used the skill of finding factors to solve polynomials shown at the basic level to attempt the problems at the proficient and advanced level. Case 4 was a scaffolded assessment and case 3 was a non-scaffolded assessment. Each of these two cases were able to find zeros at the proficient and advanced levels from factors that were found either by factoring or by using a grouping method. The scaffolds put in place for case 4 did not seem to make much difference as the student used the different skills needed throughout the different performance levels rather than the individual scaffolds given in each question. Cases 3 and 4 highlight the bridges built between the different performance level descriptors regardless of scaffolds within the questions.

In each of the cases discussed the work of the students suggested that they used the progression of skills presented at each performance level to further show their mastery. Basing the questions off of the performance level descriptors created bridges between the different questions that allowed the students to progressively build in showing their mastery of the given standard. Each student in the cases discussed were able to show mastery at the basic level, regardless of scaffolding within the exam, and used the skill of finding zeros based on factors or x- intercepts at the proficient and advanced level. The build up between questions based on the performance level descriptors suggests that individual scaffolding within questions is not needed as the different performance levels provides sufficient scaffolding for students to show their understanding.

## Separation in Scaffolding

Case 4 was a scaffolded assessment in which the student would often solve the different parts of the question individually rather than realizing the connection between the different parts. The different parts within each question were put in place as scaffolds to further aid in students correctly solving the problem. The student in case 4 often would not apply the prior parts of the question to find the correct solution, but instead solved each part individually. For example, in problem 19 the students are asked to divide the polynomial so that it is factorable and therefore easier to find the solutions. Rather than seeing the connection between the division and the factoring of the quotient, the student attempted to factor the dividend when asked.  Although the problem was scaffolded to aid in stating the steps students would need to apply in order to correctly solve, the student created a separation within the scaffolding and was not able to see the connection between the different parts.

Case 1 was also a scaffolded assessment in which the student did not appear to recognize the connection between parts of each question. The student in case 1 often did not attempt problems. The problems the student would attempt would often be singular part in the middle of a question, hindering their ability to see the connection between the different parts. Based on the evaluation, the student only attempted parts of questions that they assumed they knew although work on prior parts was necessary for the correct solution to be identified. The lack of effort throughout each question created a separation between the scaffolding and therefore created a lack of connection between the mathematical ideas.

After evaluating the scaffolded assessments there seemed to be a large disconnect between mathematical ideas. The scaffolding within the questions seemed to put more work on the students. The increase of work had students missing the connection between the different parts of each question and therefore creating more of a separation between the scaffolding. In all, scaffolding within each question seemed to create more work that either had students struggling to solve or unmotivated to even attempt.

**Connecting the Skills**

Comparing the student work on scaffolded and non- scaffolded assessments was inconclusive if students benefit from questions within an assessment being scaffolded. From the two scaffolded assessments that were analyzed it did not seem that the students identified the scaffolds within each question as an aid to solve the problem. In case study 4, the student did not connect the sequential scaffolds within each problem and seemed to solve the progressing parts individually without recognizing how each part connected. Case study 2 was also a scaffolded assessment in which the student did not attempt the majority of the steps provided for scaffolding. The scaffolding within each question seemed to create more work for the students rather than building better connections. For example, in case four the student did not connect the different parts of each question but instead solved the different parts individually.

The build up between questions based on the performance level descriptors created better connections for students. Each student that attempted questions at the basic level was able to find the correct solution. Although there was a direction error at the proficient level, each student who attempted the problem was able to correctly work through the problem based on their previous answers. Using the skills to complete questions at the basic and proficient level, all students were able to show understanding at an advanced level. The work shown within each case study had a skill used in a prior problem shown as students progressed through the assessment. By building the questions based on the performance level descriptors for each assessment, students were scaffolded based on skill.

Looking at each case study individually and comparatively it could be concluded that students were able to highlight their understanding of a given standard when questions progressively increased in difficulty based on the Wyoming state performance level descriptors. In each case where the student attempted the problem it was shown that most students were able to complete questions at the basic level with mastery. Building off the basic level, students were able to apply these skills at a proficient level. Although the misunderstanding of directions hindered results, the mathematical process shown by all students who attempted problems at the proficient and advanced demonstrated partial mastery. In all, the scaffolding of skill level showed to create the most connections between problems and allow for students to demonstrate their mathematical understanding to their highest ability.

# Chapter 5

## Conclusion and Implications

Looking at the five case studies there were many similarities. The evaluation of each case suggested that there was a lack of understanding in directions. On question 19 there were several mistakes on each of the five assessments and all the mistakes seemed to stem from the students using the incorrect expression as they proceeded through the remaining parts of the questions. Question 22 also highlighted a lack of understanding in directions as three of the cases examined had solutions that did not include integers as stated in the directions. If I were to want to evaluate students again, I would include clearer directions for these two questions to assess if it created better connections for students to obtain the correct solutions.

Another similarity between the assessments analyzed was the lack of connections between the different parts of each question. On the scaffolded assessments in cases 4 and 2, students did not seem to be able to connect their solutions from one part to the next in order to correctly identify the final solution. In case 4, the student did not use their solutions from prior parts to aid them in the remaining parts. The student instead seemed to look at each part of the question as an individual piece rather than a link between one another. In case 2 the student did not put forth effort for several parts and therefore was not able to see connections between the scaffolds as they lacked an attempt on several of the problems. Case 5, a non- scaffolded assessment, was the only case that seemed to understand the scaffolding between questions as they progressive got more challenging and use skills from previous questions to help answer other questions as they progressed through their non- scaffolded assessment.

Although there were several errors shown to be a commonality between the five assessments, there was also a similarity between mastered skills. Each assessment, besides case 5, included students synthetically dividing correctly or with minor errors. It was intriguing to see that regardless of the assessment given, most students could correctly complete a skill that was essential to further mastery of the assessed standard. The evaluation of the assessments as a whole highlighted the strengths and weaknesses of each individual student as well as the course.

## Implications

Looking in depth at each process of the students allowed me as a teacher to recognize the different topics and skills that the students need further instruction on. As I analyzed each case, I often found myself thinking of better ways to connect the different skills in order to further aid each student to show mastery at the advanced level. I also questioned if students struggled to connect the different parts and skills of each question because of lack of exposure. In future research, I would like to analyze students as done in this study after exposing them to progressively increasing difficulty of a standard using the performance level descriptors within instruction.

Assessing the different cases also had me questioning if students naturally scaffold. Case 5 was a non- scaffolded assessment in which it could be seen how the student used skills and ideas from prior problems to help solve more challenging questions. The more challenging questions on the non- scaffolded assessment did not offer much aid in the directions. The lack of aid within the non- scaffolded assessment had me questioning if students naturally scaffolded or if the progression of challenge based on the performance level descriptors throughout the assessment provided aid for the student to scaffold.

In a whole, the evaluation of the different cases showed the mathematical understanding of Algebra 2 students. The evaluation was able to highlight the strengths and weaknesses of the students by focusing on the process each student used to solve. Although the analyzing of the students solving process helped show their understanding, it also showed a lack of connection between mathematical ideas. The lack of connection between mathematical ideas hindered students as they were not able to apply these ideas to an authentic task at the advanced level. The lack of connection created a road block as students could not connect to a problem with real-world implications. From this research I can conclude the importance of analyzing the process students use to solve as well as the importance of exposure of solving with scaffolds within instruction prior to assessments. I would like to build on this research to see if students can better show their understanding on an assessment that uses performance level descriptors of a given standard after they have been exposed to similar scaffolding of problems during instructional time. In the future, I would also like to use the implications of this study to further research if students naturally scaffold. The evaluation of the different cases provided a clearer understanding of the students as well as offering a stepping stone to build future research ideas upon.

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# Attachment A

## Unit 3 Scaffolded Assessment

Algebra 2 Name:______________________________

Unit 3s Exam Date:_________________Period:________

**1.** For $x^3 + 2x^2 + 7x^4 - 3$, which of the following statements is true?

A. The degree of the polynomial is $3$.

B. Written in standard form, the polynomial is $7x^4 + x^3 + 2x^2 - 3$.

C. The polynomial is a binomial.

D. The leading coefficient is 1.

**2.** Which of the following is NOT a factor of $(x^4 - x^3 - 7x^2 + x + 6)$?

A. $x - 1$

B. $x + 2$

C. $x - 3$

D. $x + 4$

**3.** What is the remainder when $f(x) = 2x^4 + x^3 - 8x - 1$ is divided by $x - 2$?

A. $-23$

B. $23$

C. $-3$

D. $3$

**4.** Approximate the maximum and minimum of the polynomial function to the nearest tenth:

$$f(x) = x^3 - 2x^2 - 3$$

A. Minimum $(1.3, -4.2)$; Maximum $(0, -3)$

B. Minimum $(2.7, -5.5)$; Maximum $(0, 4)$

C. Minimum $(2, 0)$; Maximum $(0, 4)$

D. Minimum $(0, -7)$; Maximum $(2.7, 2.5)$

**5.** Write a polynomial in **factored form** of least degree with the given roots: $x = 0$, and $x = 3$, multiplicity of $2$.

A. $f(x) = x(x + 3)$

B. $f(x) = x^2(x + 3)^2$

C. $f(x) = x(x-2)^3$

D. $f(x) = x(x-3)^2$

**6.** Find the zeros of the function $f(x) = x^3 - 2x^2 - 19x + 20$, and describe the behavior of the graph at each zero.

A. The graph bounces off the $x$-axis at 4, and it crosses the $x$-axis at $-1$ and $-5$.

B. The graph bounces off the $x$-axis at $4, -5$, and $1$.

C. The graph crosses the $x$-axis at $5, 1,$ and $-4$.

D. The graph bounces off the $x$-axis at $-4$, and it crosses the $x$-axis at $1$ and $5$.

**7.** Factor the following polynomial: $2\mathrm{x}^3 - 54$

A. $2(x-3)(x^2+3x+9)$

B. $2(x+3)(x^2-3x+9)$

C. $2(x-6)(x^2+6x+36)$

D. $2(x+6)(x^2-6x+36)$

**8.** How does the graph of the function $f(x) = x^3 - 4$ differ from the graph of its parent function?

A. The graphs are the same.

B. Subtracting 4 translates the graph down 4 units.

C. The leading coefficient, 1, stretches the graph vertically.

D. Subtracting 4 translates the graph up 4 units.

**9.** Given the equation: $2x^3 + 6x^2 - 36x = 0$, find ALL solutions.

A. $x = -36, 2, 6$

B. $x = -6, 3$

C. $x = -3, 0, 6$

D. $x = -6, 0, 3$

**10.** If $x = 5i$ and $x = \sqrt{3}$ are zeros of a polynomial graph, then __________ and ___________ must also be zeros.

**11.** Simplify:

$(2x^3y^2 + 3x^2y - xy) + (8x^2y + 24xy + 4)$

**12.** Simplify:

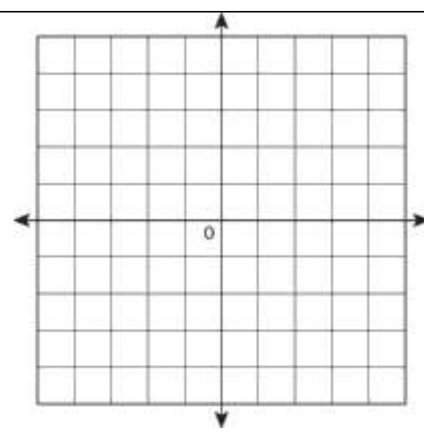
$(2x^3 - 3x + 2) - (2x^2 + 6x - 1)$

**13.** Multiply: $(x^2 - 2)(2x^2 + 5x - 3)$

**14.** Multiply: $(x + 2)^3$

**15.** Is the function $f(x) = 2x^5 + 4x^3 + 6$ odd, even, or neither? Explain your answer.

**16.** Sketch the graph of the polynomial function $f(x) = x^3 + 4x^2 + 3x$.




Use the word bank to complete the following:

| **Word Bank** | |
|---|---|
| Increasing | Decreasing |
| Positive | Negative |

**a)** $f$ is ______________________ on the intervals $(-\infty, -3)$ and $(-1, 0)$.

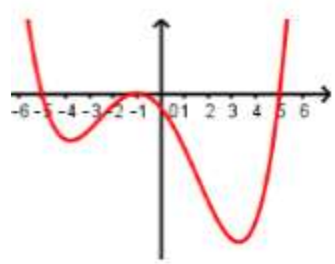
**b)** $f$ is ______________________ on the intervals $(-\infty, -2)$ and $(-0.5, \infty)$.

**c)** $f$ is ______________________ on the intervals $(-3, -1)$ and $(0, \infty)$.

**d)** $f$ is ______________________ on the interval $(-2, -0.5)$.

**17.** Identify the zeros of the polynomial function $f(x) = x(x-7)(x+7)$.

**18.** Using the graph, identify the zeros of the polynomial function.




**19.**

**a)** Divide $(x^3 - 4x^2 - 11x + 30)$ by $(x-5)$

**b)** Factor the quotient.

**c)** Write the polynomial $(x^3 - 4x^2 + 11x + 30)$ in factored form.

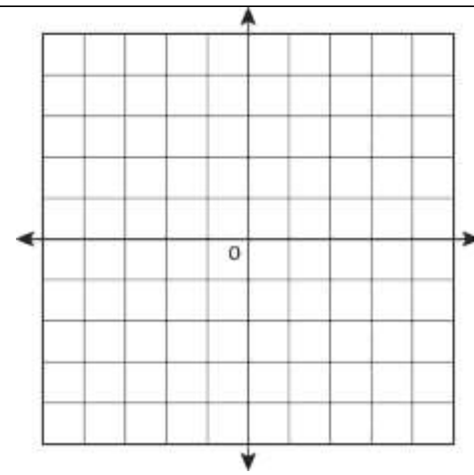
**d)** Identify the zeros of the polynomial

**e)** Identify the degree of $f(x)$ and the corresponding end behavior.

Degree:________________ As $x \to -\infty, f(x) \to$ ____________

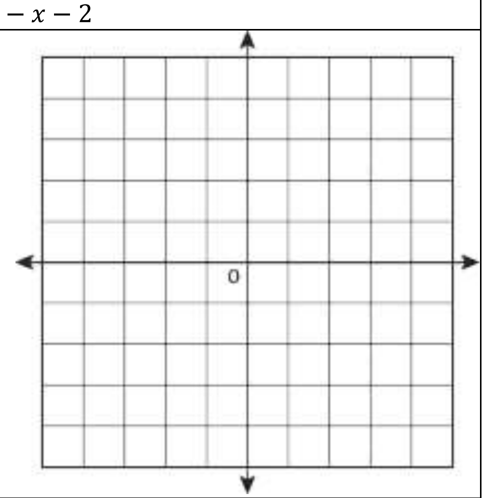
Leading Coefficient:________ As $x \to +\infty, f(x) \to$ ____________

**f)** Use the zeros to construct a sketch of the polynomial $f(x)$.




**20.** Let $P(x) = 15x^3 + 16x^2 - x - 2$

**a)** Graph $P(x)$ on the coordinate plane provided




**b)** State the degree: __________

**c)** State the number of zeros: __________

**d)** Use mathematical notation to describe the end behavior.

a. As $x \to -\infty, P(x) \to$ __________

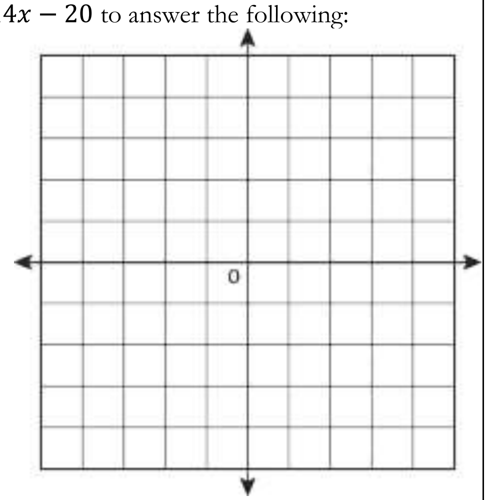
b. As $x \to +\infty, P(x) \to$ __________

**e)** State the $y$-intercept: __________

**f)** Determine all the zeros of $P(x)$ in exact form (no decimals). Show algebraic work to justify the answer.

**21.** Use the given the equation: $f(x) = x^3 - 4x^2 + 14x - 20$ to answer the following:

**a)** Find a real zero by graphing the function. **Label** the zero.

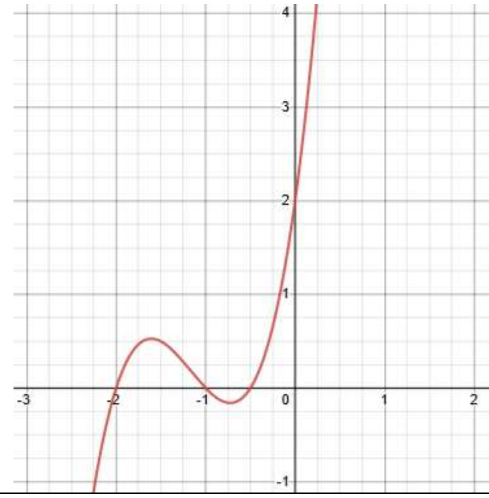



**b)** Divide the equation by the zero found from the graph to find the remaining factor(s).

**c)** Find ALL solutions to the equation.

**22.** A fireproof safe is a rectangular prism. The graph of the volume is represented below.

**a)** **Label** each point the graph intersects either the x- or y-axis.




**a)** Using the zeros of the function, write an equation in **factored form** to represent the graph. (Factors must include only **integer** values)

**b)** Write an equation in **standard form** to represent the fireproof safe.

**c)** Determine the dimensions of the safe if the volume is $270\ in^3$.

# Attachment B

## Unit 3 Unscaffolded Assessment

Algebra 2 Name:______________________________

Unit 3u Exam Date:________________Period:_______

**1.** For $x^3 + 2x^2 + 7x^4 - 3$, which of the following statements is true?

A. The degree of the polynomial is $3$.

B. Written in standard form, the polynomial is $7x^4 + x^3 + 2x^2 - 3$.

C. The polynomial is a binomial.

D. The leading coefficient is 1.

**2.** Which of the following is NOT a factor of $(x^4 - x^3 - 7x^2 + x + 6)$?

A. $x - 1$

B. $x + 2$

C. $x - 3$

D. $x + 4$

**3.** What is the remainder when $f(x) = 2x^4 + x^3 - 8x - 1$ is divided by $x - 2$?

A. $-23$

B. $23$

C. $-3$

D. $3$

**4.** Approximate the maximum and minimum of the polynomial function to the nearest tenth:

$$f(x) = x^3 - 2x^2 - 3$$

A. Minimum $(1.3, -4.2)$; Maximum $(0, -3)$

B. Minimum $(2.7, -5.5)$; Maximum $(0, 4)$

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D. Minimum $(0, -7)$; Maximum $(2.7, 2.5)$

**5.** Write a polynomial in **factored form** of least degree with the given roots: $x = 0$, and $x = 3$, multiplicity of $2$.

A. $f(x) = x(x + 3)$

B. $f(x) = x^2(x + 3)^2$

C. $f(x) = x(x - 2)^3$

D. $f(x) = x(x-3)^2$

**6.** Find the zeros of the function $f(x) = x^3 - 2x^2 - 19x + 20$, and describe the behavior of the graph at each zero.

A. The graph bounces off the $x$-axis at 4, and it crosses the $x$-axis at $-1$ and $-5$.

B. The graph bounces off the $x$-axis at $4, -5,$ and $1$.

C. The graph crosses the $x$-axis at $5, 1,$ and $-4$.

D. The graph bounces off the $x$-axis at $-4$, and it crosses the $x$-axis at $1$ and $5$.

**7.** Factor the following polynomial: $2\mathrm{x}^3 - 54$

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D. $2(x+6)(x^2-6x+36)$

**8.** How does the graph of the function $f(x) = x^3 - 4$ differ from the graph of its parent function?

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**11.** Simplify:

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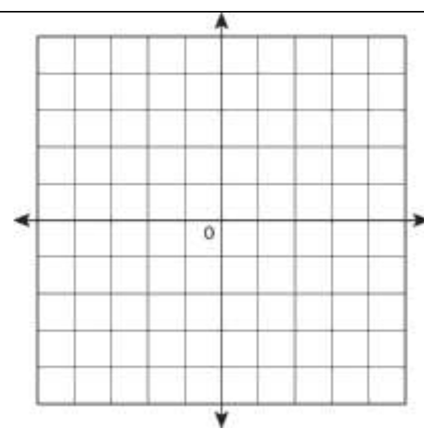


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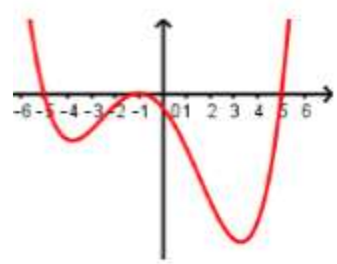
**17.** Identify the zeros of the polynomial function $f(x) = x(x-7)(x+7)$.

**18.** Using the graph, identify the zeros of the polynomial function.




**19.**

**a)** Divide $(x^3 - 4x^2 - 11x + 30)$ by $(x-5)$

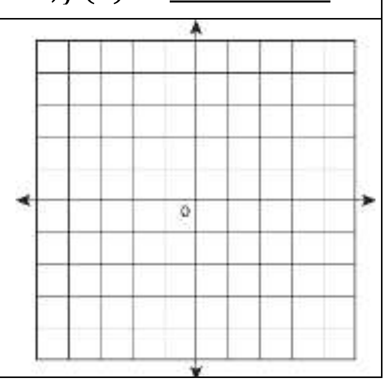
**b)** Identify the zeros of the polynomial

**c)** Identify the degree of $f(x)$ and the corresponding end behavior.

Degree:________________ As $x \to -\infty, f(x) \to$ ____________

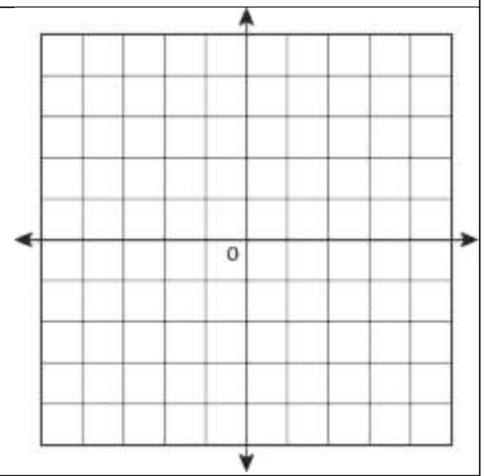
Leading Coefficient:________ As $x \to +\infty, f(x) \to$ ____________

**d)** Use the zeros to construct a sketch of the polynomial $f(x)$.



**20.** Let $P(x) = 15x^3 + 16x^2 - x - 2$

**a)** Graph $P(x)$ on the coordinate plane provided




**b)** State the degree: ___________

**c)** State the number of zeros: ___________

**d)** Use mathematical notation to describe the end behavior.

c. As $x \to -\infty, P(x) \to$ ___________

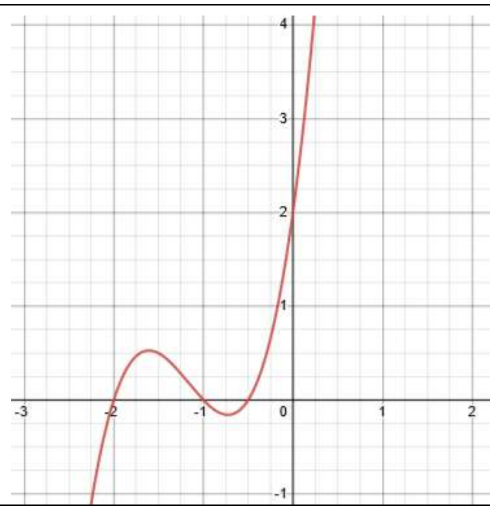
d. As $x \to +\infty, P(x) \to$ ___________

**e)** State the $y$-intercept: ___________

**f)** Determine all the zeros of $P(x)$ in exact form (no decimals). Show algebraic work to justify the answer.

**21.** Use the given the equation: $x^3 - 4x^2 + 14x - 20 = 0$, find ALL solutions:

**22.** A fireproof safe is a rectangular prism. The graph of the volume is represented below.




**b)** Using the zeros of the function, write an equation in **factored form** to represent the graph. (Factors must include only **integer** values)

**c)** Determine the dimensions of the safe if the volume is $270\ in^3$.