

Greater Yellowstone Ecosystem grizzly bear recovery and managing mortality

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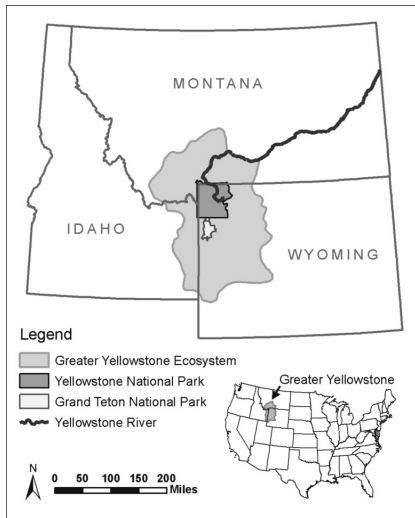
UW-NPS Research Station Harlow Summer Series
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First, a big thank you...

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- John Tschirhart Graduate Scholarship in Bioeconomics
- Carlton R. Barkhurst Dissertation Fellowship
- *UW Biodiversity Institute*: 2020 Research Enhancement Grant
- *UW Haub School of Environment and Natural Resources*:
Student Research and Creative Activities Grant

Setting: The GYE



Map: Cross et al. (2012)



Map: Lisa Landenburger, USGS

Species: Grizzly Bears

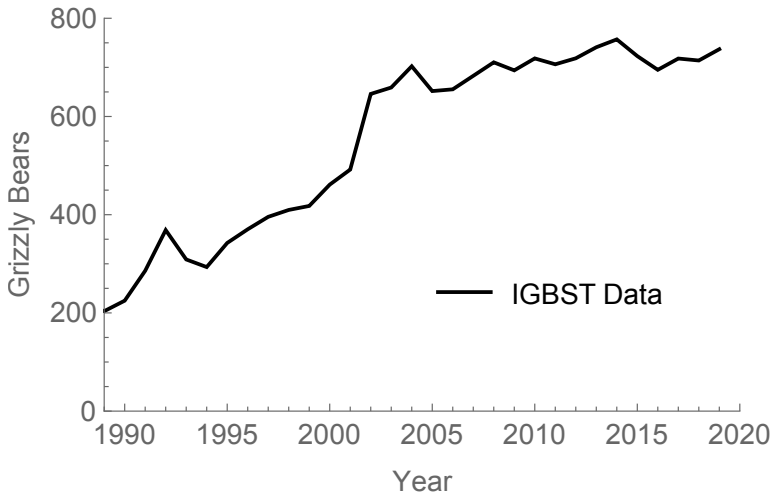


Photo: Ray Paunovich



Photo: Interagency Grizzly Bear Study Team

Grizzly bears over time...



And the problem...

AP **Montana euthanizes 2 grizzly bears after property damaged** Top Stories

Montana euthanizes 2 grizzly bears after property damaged

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Grizzly bear killed by wildlife officials after killing cow

Associated Press Aug 4, 2020 Updated Aug 14, 2020 🔥 0

And the problem...

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Montana hunter reports shooting grizzly bear in self-defense

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Man injured in grizzly attack east of Dubois; bear killed

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Cody man injured in grizzly attack

May 1, 2020 Updated Jun 6, 2020 📌 0

Man injured in grizzly attack east of Dubois; bear killed

Joshua Wolfson May 18, 2020 Updated Jun 23, 2020 📌 0

And the problem...

AP
USA TODAY
COVID-19 Comparing vaccines
HERE COME THE CICADAS Tracking Brood X
COVID-19 BY STATE Track vaccinations

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NATION NOW

Guide fatally mauled by bear while helping Florida hunter in Wyoming

AP

Griz

Ashley May USA TODAY

Published 8:20 a.m. ET Sep. 16, 2018 | Updated 4:50 a.m. ET Sep. 17, 2018

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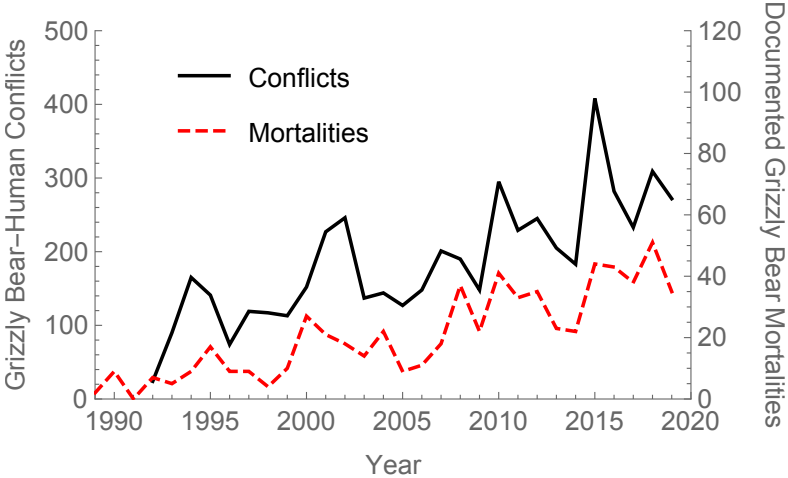
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May 1, 2020 Updated Jun 6, 2020 0

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Over time...



Data: Interagency Grizzly Bear Study Team

Average...

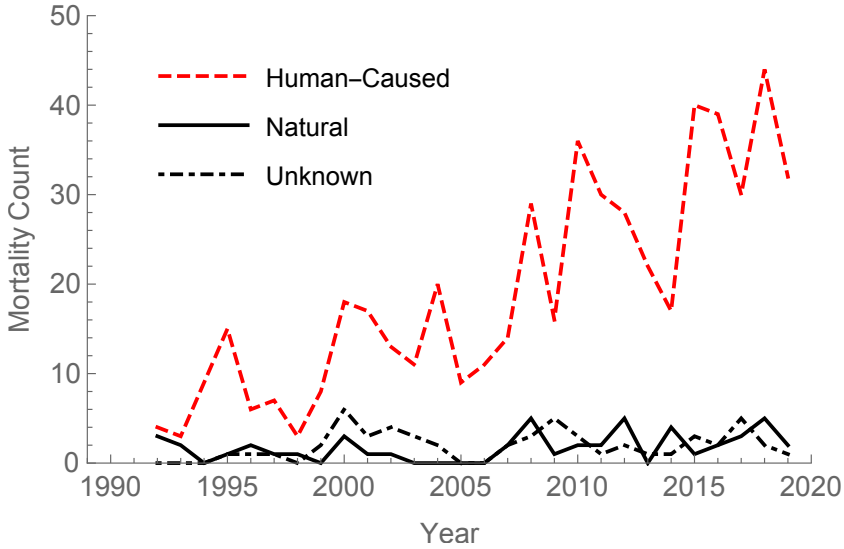
- Conflicts

Conflict Type	Average Annual Count	Average Annual Share
Livestock depredations	157	59%
Unnatural foods	61	23%
Property damages	41	15%
Human injuries	7	2%
Total annual	266	100%

- Bear mortality

Mortality Type	Average Annual Count	Average Annual Share
Management	19	54%
Self-Defense	10	28%
Vehicular	3	8%
Other	2	6%
Hunter-Mistake	2	4%
Total annual	36	100%

Grizzly bear mortalities over time...



Data: Interagency Grizzly Bear Study Team

Roadmap

Motivation

- Controversy abounds
 - Should grizzly bears be delisted from the ESA?
 - Should state agencies enact grizzly bear harvests?
- Humans and grizzly bears are *inextricably* linked
 - High-quality ecological data indicates population recovery...
 - But that hasn't swayed people's opinions. Why?

Step 1 - Exploratory Bioeconomic Analysis

- Use existing data
- Examine socially optimal grizzly bear management

Step 2 - Nonmarket Valuation

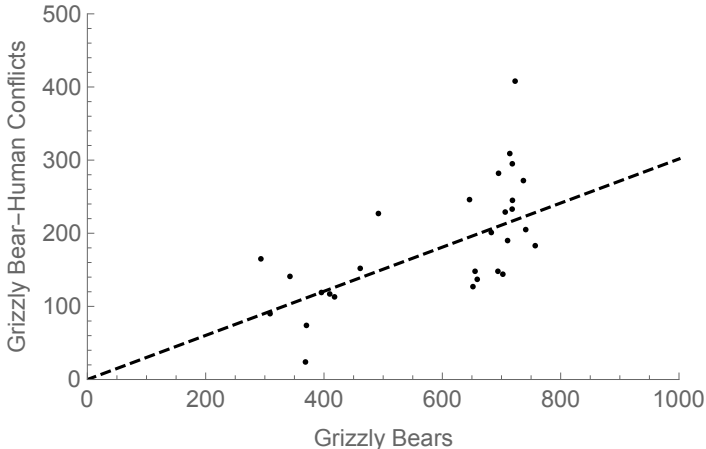
- Fill in gaps in the existing data
- Design and administer a nonmarket valuation survey
- Quantify nonmarket benefits and damages from grizzly bears

Step 1 - Bioeconomic Analysis

- **Goal:** Examine how grizzly bear management affects social welfare
 - Simultaneously account for various stakeholder groups
 - Compare outcomes under alternative management strategies
 - “*Reactive*” → management as is (i.e., no harvests)
 - “*Active*” → protections removed (i.e., harvests)
- **Tool:** Dynamic bioeconomic model
 - Model stock-dependent ecological/economic relationships
 - Account for rigid and inflexible nature of future grizzly bear policies
 - Replicate “on/off” management lever
- **Findings:**
 - The value of a live grizzly bear in the wild is negative and decreasing
 - Compared to reactive management, active management leads to...
 - Fewer grizzly bears, conflicts, *and* mortalities
 - Larger (and non-negative) value per live grizzly bear
 - Greater discounted social net benefits
 - The results are sensitive to key ecological and economic parameters

Quantifying Links to Humans...

- **More bears → more conflicts**



- **Assumptions:**

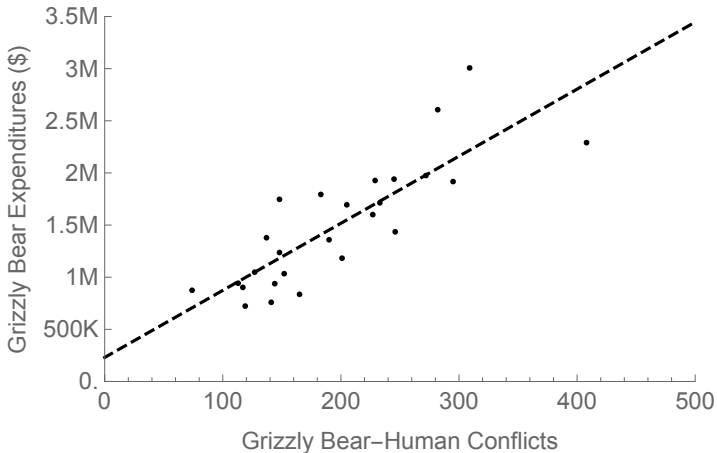
$$I(G), I'(G) > 0, I''(G) = 0$$

Mortalities

Negative Binomial Regressions

Quantifying Links to Humans...

- **More conflicts** → **more damages**



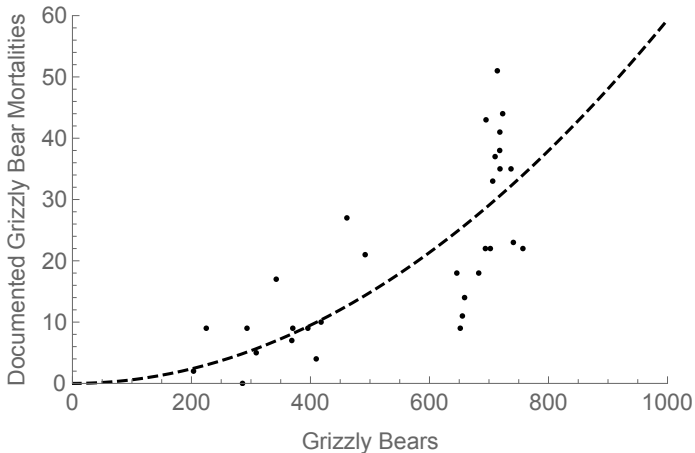
- **Assumptions:**

$$D(I), D'(I) > 0, D''(I) = 0$$

OLS Regressions

Quantifying Links to Ecology...

- **More bears** → **more mortalities**



- **Assumptions:**

$$M(G), M'(G) > 0, M''(G) < 0$$

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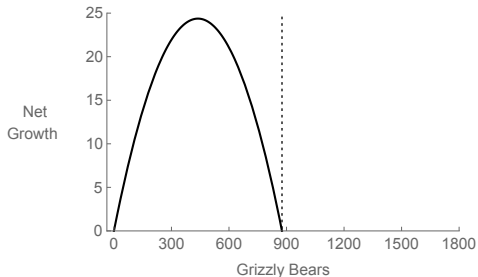
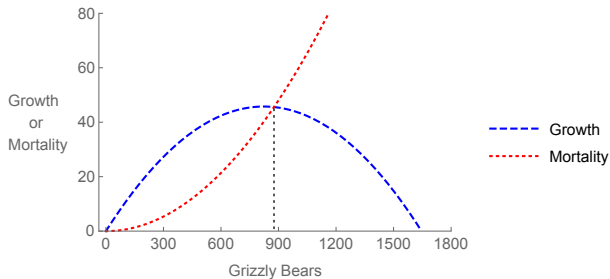
Negative Binomial Regressions

Links to Ecology: Bear Population Growth

$$\text{change in } G \text{ over time} = \overbrace{\text{Logistic growth} - \text{Harvest}}^{\text{Standard}} - \text{Mortality}$$

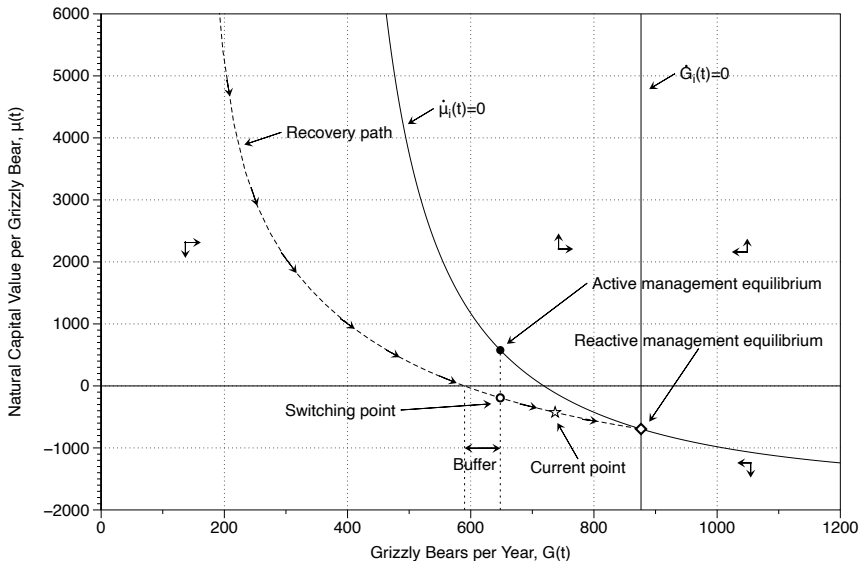
- Logistic growth: $L(G)$
 - reflects density dependence
- Non-harvest mortality: $M(G)$ also depends on...
 - Expansion of grizzly bear range on private land (PVT, \uparrow)
 - Food resource quality \rightarrow whitebark pine (WBP, \downarrow)
 - Climate \rightarrow Palmer Drought Severity Index (PDSI, \uparrow)

Population Growth: Fitted Functions



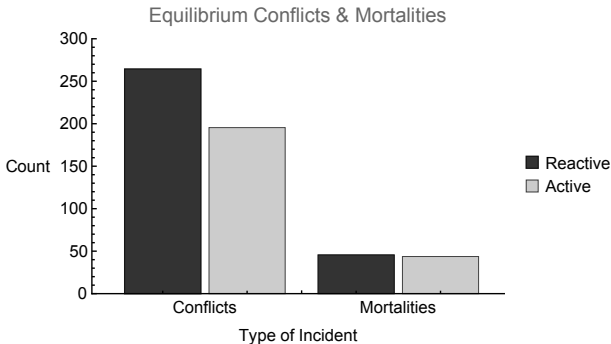
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Results - Phase Plane

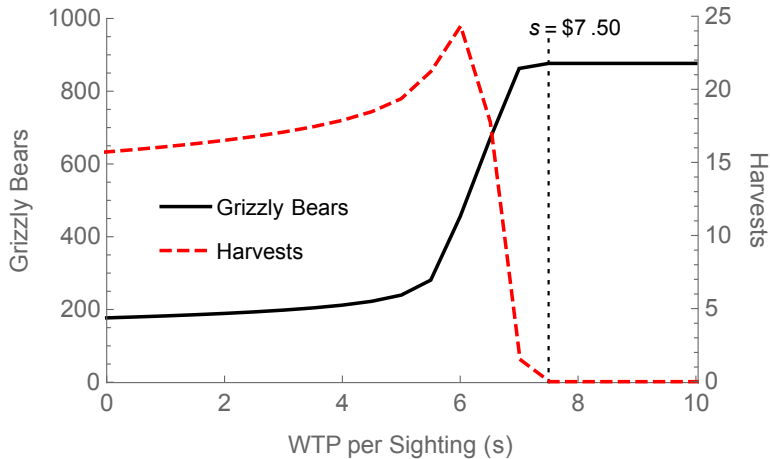


Results - Management Outcomes

Management Strategy	Discounted Social Net Benefits (\$)
Active	1,928,482,102
Reactive	1,920,168,145
Difference	8,313,957

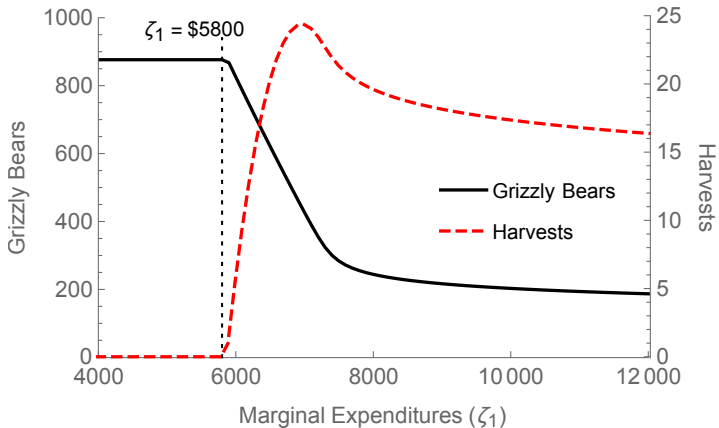


Sensitivity Analysis - Benefits



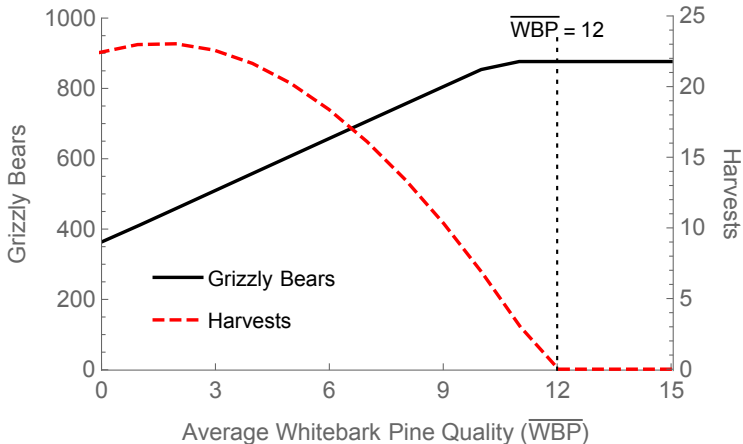
- Baseline WTP = 6.45

Sensitivity Analysis - Damages



- Baseline $\zeta_1 = 6432$

Sensitivity Analysis - Ecology



- Baseline $\overline{WBP} = 5.8001$

See more plots:

Nonuse Value

Consumptive Use Value

Discount Rate

Step 1 - Summary

Baseline parameterization

- Compared to reactive management, active management leads to...
 - Fewer grizzly bears, conflicts, and total mortalities
 - Larger natural capital value per grizzly bear in the wild
 - Greater social net benefits

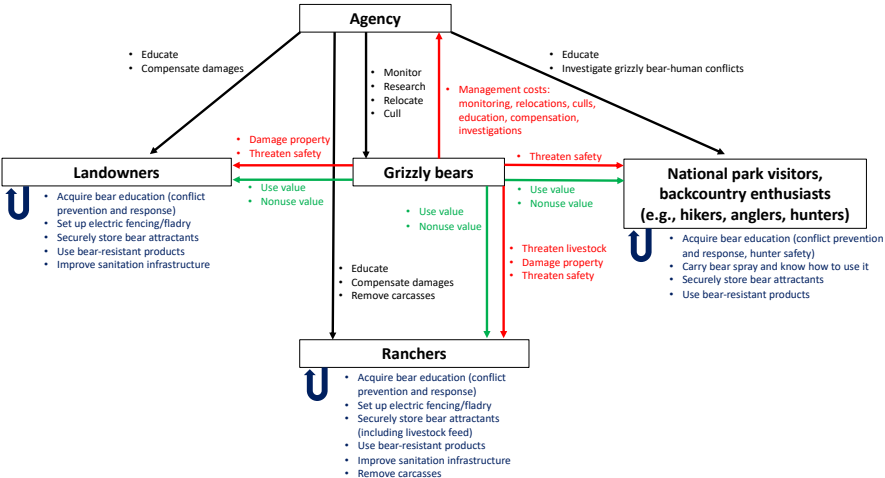
Sensitivity analysis

- High uncertainty in parameter estimates
- Thresholds at which optimal strategy switches for key parameters:
 - WTP per grizzly bear sighting
 - Marginal expenditures per grizzly bear-human conflict
 - Average whitebark pine quality

Conclusion

- Federal protections may be having unintended effects
 - If removed, managers could transform resource from liability to asset
- Need better understanding of links b/w grizzly bears and humans

Step 2 - Drill into the Web of Bear-Human Interactions



Nonmarket Valuation

- **Goal:** Quantify “nonmarket” grizzly bear benefits & damages
 - Explore use and non-use values among locals and non-locals
- **Tool:** Nonmarket valuation
 - Stated preference methods → choice experiments
- **Findings:**
 - People are willing to pay for changes in grizzly bear conditions
 - WTP estimates differ among locals and non-locals (in surprising ways)

Background

Nonmarket valuation

- People benefit from ecosystem goods not traded in markets
- Externalities from such goods are a source of market failure
- Wedge between market and socially desirable outcomes
 - Economic policies can reduce the wedge, but...
 - Information on nonmarket benefits & damages is needed

Stated preference (SP) methods

- People state how much they would be willing to pay
 - No need to observe actual behavior
 - Only known method to quantify WTP for nonuse values
- Changes to be valued are hypothetical

Survey Implementation

Survey mode

- Online mode
- Qualtrics survey software

Sample

Sample vs. Population Means

- Residents of Wyoming, Montana, and Idaho
- Gender-balanced panel

Pilot survey

- November 19 - 29, 2020
- $N = 350$
- Goal: refine attribute level labels

Final survey

- December 6, 2020 - January 17, 2021
- $N = 2,479$

Willingness to Pay

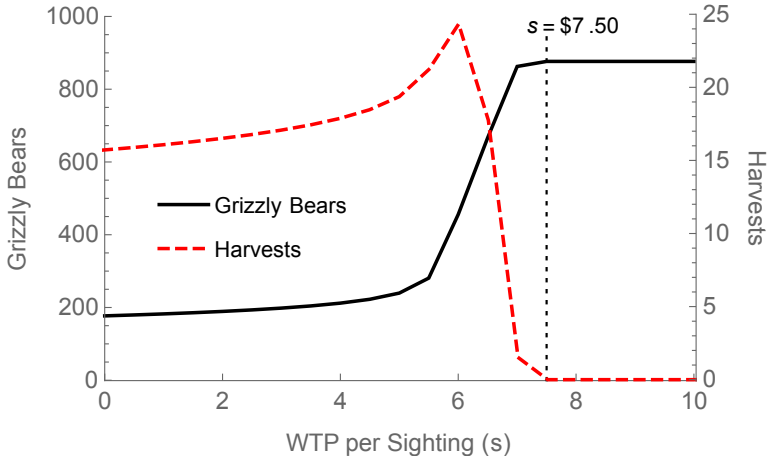
Willingness to Pay

Att.	CE	WTP	Weighted WTP	Units (\$ per...)
S	1	<i>Non-locals:</i> 0.31	0.29	% increase
		<i>Locals:</i> 0.21		
R	1	<i>Non-locals:</i> 9.75	9.83	1 fewer attack per mill. visitors
		<i>Locals:</i> 10.23		
C	2	<i>Non-locals:</i> 0.25	0.23	1 fewer conflict
		<i>Locals:</i> 0.15		
E	2	<i>Non-locals:</i> 10.86	12.63	% decrease
		<i>Locals:</i> 20.55		

- **Compared to non-locals, locals value..**
 - Increases in sightings less but decreases in risk more
 - Decreases in conflicts less but decreases in extinction probability more

S = chance of park sighting | R = risk of park injury | C = annual grizzly bear-human conflicts | E = 100-year grizzly bear extinction probability

Return to Benefits Sensitivity Analysis



- If extrapolate result for S to a guaranteed sighting, $wtp \gg 7.50...$

Step 2 - Summary

Issue

- Grizzly bear management has become highly controversial
- Missing puzzle piece: linkages b/w grizzly bears and humans

Findings

- Both locals and non-locals are willing to pay to...
 - Increase chance of park sighting
 - Decrease risk of park injury, conflicts, and extinction probability

Future work

- Refine the approaches and merge the work
- Conduct outreach!



Questions?



Photo: Ray Paunovich

Conflict Negative Binomial Regressions

	Linear	Squared	Unconstrained
lnG	1 (-)	2 (-)	0.78*** (3.93)
PVT	0.00057*** (2.70)	0.00042 (1.34)	0.00061*** (3.18)
WBP	-0.018** (-2.21)	-0.015 (-1.08)	-0.020*** (-2.61)
PDSI	0.0091 (0.40)	0.044 (1.20)	0.0027 (0.11)
Constant	-1.32*** (-11.49)	-7.53*** (-41.56)	0.040 (0.03)
α	0.088	0.216	0.084
Rb. Std. Err.	0.036	0.063	0.039
N	27	27	27
Corr ²	0.641	0.544	0.654

z-scores in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Expenditure OLS Regressions

	Linear	Squared
CONF	6,432.6*** (5.34)	10,644.6*** (2.87)
CONF ²		-9.30 (-1.14)
Constant	230,397.8 (1.08)	-186,208.8 (-0.48)
<i>N</i>	26	26
<i>R</i> ²	0.686	0.700

t-statistics in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Take me back!

Parameterization - Ecology

$$\dot{G} = \overbrace{L(G(t)) - y(t)}^{\text{Standard}} - \underbrace{M(G(t))}_{\text{Mortality}}$$

Logistic
Harvest
Mortality

- Logistic growth function: $rG(t) \left(1 - \frac{G(t)}{K}\right)$
 - r is the intrinsic rate of growth, K is the logistic limit
 - Assumptions: $L(0) = L(K) = 0$, $L'(0) = r$, $L''(G) < 0$
- Non-harvest mortality is a function of...
 - Expansion of grizzly bear range on private land (PVT)
 - Food resource quality → whitebark pine (WBP)
 - Climate → Palmer Drought Severity Index (PDSI)

$$M(G) = M(G, PVT, PDSI, WBP)$$

$$= e^{\alpha_0 + \alpha_1 \ln(G) + \alpha_2 PVT + \alpha_3 PDSI + \alpha_4 WBP + \varepsilon}$$

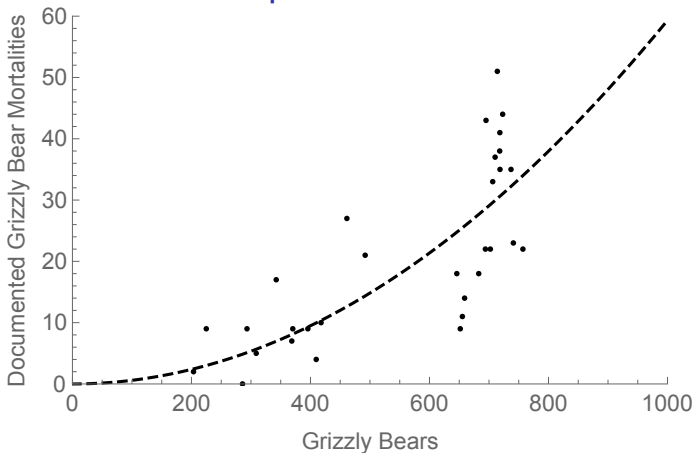
Mortality Negative Binomial Regressions

	Linear	Squared	Unconstrained
InG	1 (-)	2 (-)	1.61*** (5.33)
PVT	0.00050** (2.19)	0.00024 (0.88)	0.00034 (1.51)
WBP	-0.035*** (-3.33)	-0.025* (-1.71)	-0.029** (-2.21)
PDSI	0.0093 (0.28)	0.025 (0.60)	0.019 (0.51)
Constant	-3.38*** (-21.59)	-9.68*** (-51.75)	-7.20*** (-3.67)
α	0.154	0.141	0.127
Rb. Std. Err.	0.064	0.050	0.049
N	28	28	28
Corr ²	0.596	0.612	0.618

z-scores in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Stock-Dependent Mortalities



- Assumptions:

$$M'(G) > 0, M''(G) > 0$$

[Negative Binomial Regressions](#)[Take me back!](#)

Parameterization - Ecology

Net growth in the current state of the world ($y = 0$) is

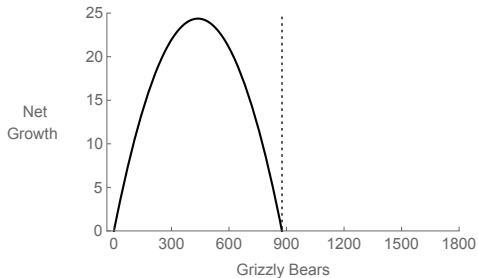
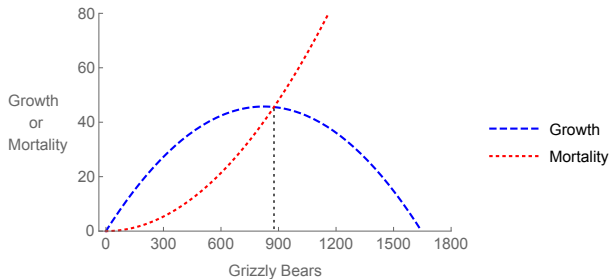
$$\dot{G} = rG(t) \left(1 - \frac{G(t)}{K} \right) - \hat{\gamma}_M G^2$$

Solving the differential equation yields

$$G(t) = \frac{e^{rt} G_0 K r}{\hat{\gamma}_M (e^{rt} - 1) G_0 K + ((e^{rt} - 1) G_0 + K) r}$$

- Parameterization steps:
 - Replace $\hat{\gamma}_M$ with $\exp(\hat{\alpha}_0 + \hat{\alpha}_2 PVT(t) + \hat{\alpha}_3 WBP(t) + \hat{\alpha}_4 PDSI(t))$
 - Set initial guesses for r and K
 - Calculate predicted population in each period using the annual data
 - Minimize sum of squared errors by optimizing r and K

Parameterization - Ecology

[Take me back!](#)

Parameterization - Nonuse Value

- US Fish & Wildlife Service (2000)
 - Interested in reintroduction of Bitterroot Ecosystem grizzly bears
 - Stated preference survey with local, regional, and national samples
 - Elicited mean lifetime contribution for grizzly bear reintroduction (w)
 - Converted to aggregate annual net economic value (n_A):

$$n_A = w \cdot H_t \cdot (2f - 1) \cdot \delta \cdot z$$

H_t = Total number of households

f = Proportion of households in favor of reintroduction

δ = Discount rate

z = Scaler to account for hypothetical nature of exercise

- I employ unit value benefit transfers
 - For example, $w = 65.45$ (2019 USD)

Parameterization - Nonuse Value

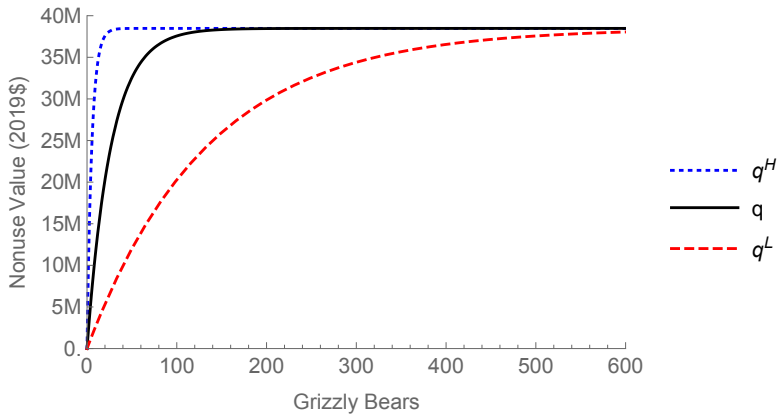
- WTP for threatened species increases at decreasing rate
 1. Loomis and White (1996)
 2. Richardson and Loomis (2009)
- I create a scaling function to link nonuse value and population size:

$$R(G) = (1 - e^{-qG})$$

- Assumptions
 - Nonuse value stops increasing after threshold (K_T) is reached
 - For $G < K_T$: $R'(G) > 0$ and $R''(G) < 0$
 - Calibrated using two data points: $R(0) = 0$, $R(K_T) = 1$
- Scaled aggregate annual nonuse value:

$$N(G) = n_A \cdot R(G)$$

Parameterization - Nonuse Value

[Take me back to Golden Rule!](#)[Take me back to Sensitivity Analysis!](#)

Parameterization - Nonconsumptive Use Value

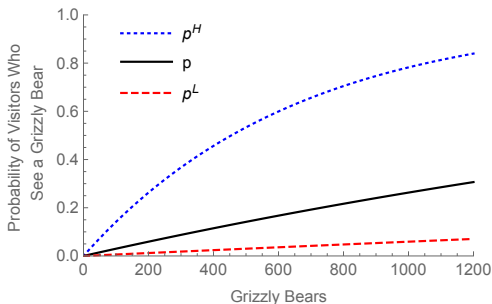
- All nonconsumptive use value comes from grizzly bear sightings:

$$S(G) = v \cdot s \cdot P(G)$$

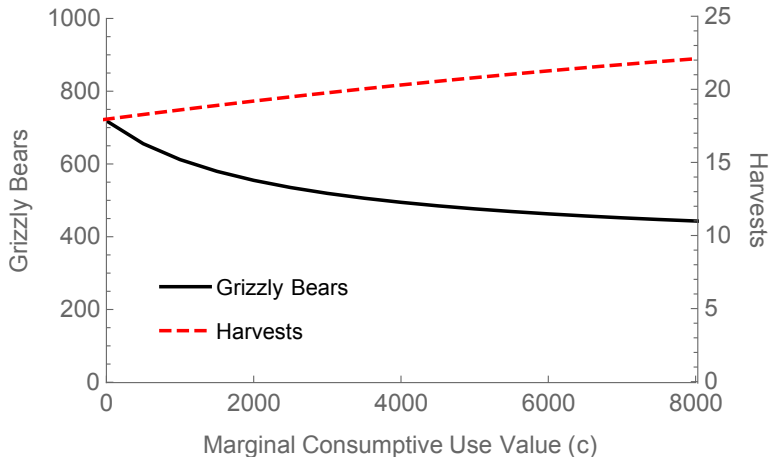
- v is the annual number of park visitors who value seeing a grizzly
 - Obtained NPS data on the number of 2019 YNP visitors
 - Scaled the above using stats from Richardson et al. (2015):
 - % of visitors who consider viewing bears to be “very important” (62%)
 - % of visitors who do not prefer to see a black bear (95%)
 - % of visitors who see a bear and stop to watch it (82%)
- s is a visitor’s WTP per grizzly bear sighting
 - Averaged the literature estimates of WTP per day of bear viewing
 - Scaled the above by the average length of a YNP bear-jam

Parameterization - Nonconsumptive Use Value

- $P(G)$ is the ratio of park visitors who see a grizzly bear
 - Data points: Duffield et al. (2006), Richardson et al. (2014, 2015)
 - Assumptions: $P'(G) > 0$, $P''(G) < 0$, $P(0) = 0$
 - Function: $P(G) = 1 - e^{-pG}$
 - Optimization of p : Microsoft Excel solver

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Sensitivity Analysis - Consumptive Use Value



Background - Total Economic Value

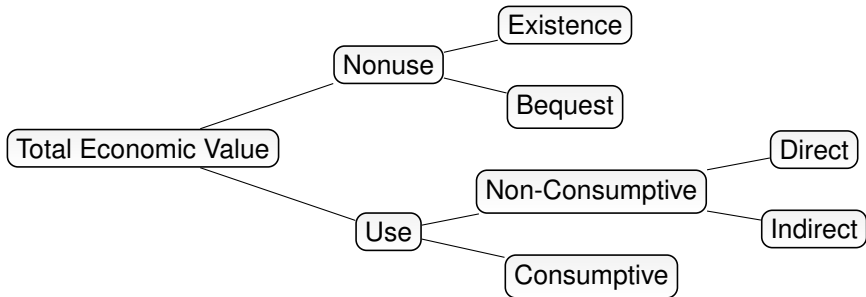


Figure: Types of values in the TEV framework.

Choice Experiment 1

	Current Levels	Proposed Levels
Park visitor's chance of seeing a grizzly bear	30%	45%
Park visitor's risk of being injured by a grizzly bear	1 in 2.70 million	1 in 1.35 million
National park entrance fee	\$35	\$45

Which would you prefer for your next recreation trip to Yellowstone and/or Grand Teton National Park?

Current levels (i.e., no changes and no increase in the entrance fee)

Proposed levels (i.e., changes and an increase in the entrance fee)

Take me back!

Choice Experiment 2

	Current Levels	Proposed Levels
Grizzly bear-human conflicts in the GYE	270 per year	210 per year
100-year extinction probability of GYE grizzly bears	2.0%	1.5%
One-time cost to your household	\$0	\$40

Which would you vote for?

Current levels (i.e., no changes and no one-time cost)

Proposed levels (i.e., changes and a one-time cost)

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Summary Statistics

Demographic	Description	State	Sample	Population
			Mean	Mean
State Residence	Proportion of all GYE state residents	ID	0.511	0.520
		MT	0.325	0.311
		WY	0.164	0.169
Sex	=1 if female	ID	0.540	0.501
		MT	0.517	0.501
		WY	0.505	0.491
		GYE	0.527	0.499
Age Category	=1 (18-24) to 7 (75 and over)	ID	3.53	3.79
		MT	3.73	3.94
		WY	3.68	3.82
		GYE	3.62	3.84
Education Category	=1 (some high school) to 5 (grad. degree)	ID	3.07	2.95
		MT	3.20	3.04
		WY	3.10	2.99
		GYE	3.12	2.99

More Summary Statistics

Demographic	Description	State	Sample	Population
			Mean	Mean
Race	=1 if white	ID	0.897	0.894
		MT	0.895	0.880
		WY	0.911	0.909
		GYE	0.899	0.892
Household Size	Number of people	ID	2.87	2.68
		MT	2.70	2.38
		WY	2.64	2.42
		GYE	2.78	2.54
Income ^m	Annual pre-tax household income (US\$10,000)	ID	5.66	6.10
		MT	6.33	5.72
		WY	5.80	6.50
		GYE	5.90	6.05

Take me back!

Population data: U.S. Census Bureau

Choice Experiment 1 - Regressions

	No Interactions		Interactions	
	<i>Logit</i> -All-	<i>Logit</i> -First-	<i>Logit</i> -All-	<i>Logit</i> -First-
Intercept	-0.192*** (0.055)	-0.094 (0.094)	-0.191*** (0.055)	-0.094 (0.094)
Sightings (S)	0.021*** (0.002)	0.016*** (0.003)	0.022*** (0.002)	0.016*** (0.003)
Risk (R)	-0.696*** (0.138)	-0.593* (0.243)	-0.690*** (0.150)	-0.571* (0.262)
Fee	-0.071*** (0.005)	-0.071*** (0.008)	-0.071*** (0.005)	-0.071*** (0.008)
S·Local			-0.007* (0.003)	-0.004 (0.005)
R·Local			-0.035 (0.335)	-0.174 (0.597)
log <i>L</i>	-4,656	-1,571	-4,653	-1,570
McFad. R^2	0.040	0.034	0.041	0.034

Standard errors in parentheses; * $p < .05$, ** $p < .01$, *** $p < .001$

Choice Experiment 2 - Analysis

Equations of interest

- Utilities:

$$U_A = \delta_1 C_A + \delta_2 E_A + \delta_3 T_A + \delta_4 (C_A \cdot L) + \delta_5 (E_A \cdot L) + \varepsilon_A$$

$$U_B = \delta_1 C_B + \delta_2 E_B + \delta_3 T_B + \delta_4 (C_B \cdot L) + \delta_5 (E_B \cdot L) + k_B + \varepsilon_B$$

- Probabilities:

$$P_A = \frac{1}{1 + e^{(\delta_1 C_B + \delta_2 E_B + \delta_3 T_B + \delta_4 (C_B \cdot L) + \delta_5 (E_B \cdot L) + k_B) - (\delta_1 C_A + \delta_2 E_A + \delta_3 T_A + \delta_4 (C_A \cdot L) + \delta_5 (E_A \cdot L))}}$$

$$P_B = \frac{1}{1 + e^{(\delta_1 C_A + \delta_2 E_A + \delta_3 T_A + \delta_4 (C_A \cdot L) + \delta_5 (E_A \cdot L)) - (\delta_1 C_B + \delta_2 E_B + \delta_3 T_B + \delta_4 (C_B \cdot L) + \delta_5 (E_B \cdot L) + k_B)}}$$

C = annual conflicts | E = 100-year extinction probability | T = vehicle registration fee | L = local

Take me back!

Choice Experiment 2 - Regressions

	No Interactions		Interactions	
	<i>Logit -All-</i>	<i>Logit -First-</i>	<i>Logit -All-</i>	<i>Logit -First-</i>
Intercept	-0.005 (0.058)	-0.154 (0.100)	-0.003 (0.058)	-0.155 (0.100)
Conflicts (C)	-0.005*** (0.000)	-0.004*** (0.001)	-0.005*** (0.000)	-0.005*** (0.001)
Extinction (E)	-0.252*** (0.049)	-0.243** (0.084)	-0.216*** (0.052)	-0.182* (0.089)
Fee	-0.020*** (0.001)	-0.014*** (0.002)	-0.020*** (0.001)	-0.014*** (0.002)
C·Local			0.002* (0.001)	0.004** (0.001)
E·Local			-0.193* (0.096)	-0.340* (0.161)
log <i>L</i>	-4,721	-1,622	-4,717	-1,617
McFad. R^2	0.050	0.031	0.051	0.034

Standard errors in parentheses; * $p < .05$, ** $p < .01$, *** $p < .001$

Stage 2: Derivation

- Duality identity: $\bar{u} = V(p, s, r, E(p, s, r, \bar{u}))$
- Differentiate both sides with respect to p and rearrange:

$$\frac{\partial E(\cdot)}{\partial p} = - \left(\frac{\partial V(\cdot)}{\partial p} \right) / \left(\frac{\partial V(\cdot)}{\partial m} \right)$$

- From Roy's Identity, we know the above is also equal to x :

$$m'(p) = \frac{\partial m(p)}{\partial p} = \frac{\partial E(\cdot)}{\partial p} = e^{\alpha + \beta_s s + \beta_r r + \beta_p p + \beta_m m + \beta_z Z}$$

- The above is a differential equation that can be solved:

$$m(p) = -\frac{1}{\beta_m} \ln \left(-\frac{\beta_m}{\beta_p} e^{\alpha + \beta_s s + \beta_r r + \beta_p p + \beta_z Z} - \beta_m c \right)$$

- With $c = u$, the above is the quasi-expend. function, $\tilde{E}(p, s, r, u)$
- Utilizing $\tilde{E}(p, s, r, u) = m$ and solving for u yields indirect utility

Stage 2: Random Utility Model

- Indirect utility before and after a policy:

$$V_{A,i} = -\frac{1}{\beta_m} e^{-\beta_m m_i} - \frac{1}{\beta_p} e^{\alpha + \beta_s s_{A,i} + \beta_r r_{A,i} + \beta_p p_i + \beta_z Z_i},$$

$$V_{B,i} = -\frac{1}{\beta_m} e^{-\beta_m m_i} - \frac{1}{\beta_p} e^{\alpha + \beta_s s_{B,i} + \beta_r r_{B,i} + \beta_p (p_i + f_i) + \beta_z Z_i}$$

- Probit model:

$$\Pr[y_i = 1] = \Pr[\varepsilon_{A,i} - \varepsilon_{B,i} < V_{B,i} - V_{A,i}] = \Phi \left[\frac{V_{B,i} - V_{A,i}}{\sigma} \right]$$

- Log likelihood function:

$$\log L = \sum_{i=1}^N y_i \cdot \log(\Pr[y_i = 1]) + (1 - y_i) \cdot \log(1 - \Pr[y_i = 1])$$

- Estimation: maximum likelihood