THE COMMONS AND THE OPTIMAL NUMBER OF FIRMS*

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The "problem of the commons" is a frequently cited example of market failure in which exploiters' pursuit of profits does not lead to the attainment of a social or Pareto optimum. In particular, a free-access equilibrium induces an unrestricted number of exploiters or firms to equate the variable input's average product, instead of its marginal product, to the input's real rental rate; hence, the rents of the variable input are driven to zero [Haveman, 1973].[1] When the number of firms in a commons is unrestricted, the scarce factor (e.g., the fishery, the hunting ground) is not imputed a rent. A social optimum can be achieved if a single firm exploits a commons and sells its output in a perfectly competitive market [Weitzman, 1974].

The purpose of this note is to derive an expression for the optimal number of firms exploiting a commons when the resulting output is sold in an imperfectly competitive market.[2] Since demand inelasticity due to monopoly power leads to overconservation, while an increase in the number of exploiting firms typically leads to underconservation, a finite number of firms for a commons can be found corresponding to a social or Pareto optimum. In particular, the optimum number of firms depends directly on the elasticity of the input productivity and inversely on the price elasticity of market demand.

I. THE OPTIMAL NUMBER OF FIRMS AND THE COMMONS

In the model here, Nash behavior is assumed to characterize the firms; that is, each firm takes the level of exploitation of others as given when determining its own exploitation rate. Hence, a

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1. On common property, see Cornes and Sandler [1983], Dorfman [1974], Dasgupta and Heal [1979], and Sinn [1984].

2. The analysis here differs in a number of important respects from previous studies of oligopolistic exploitation of a commons. First, previous investigations *fixed* the number of firms. See, for example, Dasgupta and Heal [1979], Kemp and Long[1980], and McMillan and Sinn[1984]. Second, the dynamic framework used by these articles required stronger supply-side assumptions than those of our static renewable-resource model. Third, most previous articles assumed isoelastic market demand; our model does not restrict market demand elasticity.

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zero conjectural variation, regarding the influence that one's own exploitation rate is expected to have on others, is assumed. The relevant industry consists of profit-maximizing firms, each of which has access to an exogenously fixed, common property renewable resource. Each firm combines a single private input with the fixed factor of the commons in order to produce a single marketable output. The firms face a downward-sloping aggregate inverse demand function $P(C)$, where P is the output price and C is the aggregate output of the commons.[3] Henceforth, we shall refer to the common property as a fishing ground, the variable input as fishing vessels, and the output as fish caught. With the size of the commons fixed, the total catch depends entirely upon the size of the total fleet R. Following Dasgupta and Heal [1979], the aggregate production function is assumed to satisfy

$$(1) \quad C = F(R), \qquad F'(R) > 0, \qquad F''(R) < 0, \qquad F(0) = 0,$$

where $F(R)$ is bounded above by the entire fish population of the commons. These assumptions imply that

$$(2) \qquad F(R)/R > F'(R) \text{ and } \lim_{R\to\infty} F(R)/R = 0.$$

To simplify the analysis still further, we assume that identical firms ply the commons and that the fish are evenly dispersed. Hence, each vessel makes the same catch over what might be called a "pure" commons. With these assumptions, each firm's catch c can be related to the total catch via the following production possibility function:

$$(3) \qquad \begin{aligned} c &= [r/(r + \tilde{R})]F(r + \tilde{R}) \\ &= (r/R)F(R), \end{aligned}$$

where r is the firm's vessels and $\tilde{R} = R - r$ is the size of the rest of the fleet which fishes the commons.

The Nash solution requires the firm to choose its fleet size to maximize profit π, while taking $\tilde{R}$ as fixed. Given (3), the firm's problem, then, is to

$$(4) \qquad \max_{\{r\}} \{[P(C)C/R - w]r\},$$

3. If no near substitutes exist whatsoever, then $P(C)$ is the entire market inverse demand function for the output produced from the commons. When, however, some outside competition exists, $P(C)$ is the commons' share of the market inverse demand function.

where w is the rental rate for vessels and $C = F(r + \tilde{R})$. This maximum is accomplished by hiring vessels up to the point where

$$(5) \quad \pi_r = \frac{P(C)C}{R} - w + \frac{r}{R}\left\{[P'(C)C + P(C)]F'(R) - \frac{P(C)C}{R}\right\} = 0,$$

provided that the second-order condition, $\pi_{rr} < 0$, is also satisfied. In (5), a subscript on π indicates a partial derivative, and primes refer to derivatives. We denote the fleet size and aggregate catch that satisfies (5) as $\hat{R}$ and $\hat{C}$, respectively. As the number of firms increases toward atomistic exploitation, the ratio r/R in (5) goes to zero owing to our assumption of symmetry, since $r/R = 1/n$. For atomistic exploitation where $n = \infty$, (5) becomes

$$(6) \qquad P(C)C/R = w,$$

indicating that the value of average product is equated to the vessel's rental rate and that profits are driven in zero. That is, the tragedy of the commons applies in full. For a single exploiter or monopolist, (5) implies that

$$(7) \qquad [P'(C)C + P(C)]F'(R) = w,$$

in which the marginal revenue product equals the rental rate.

The Pareto or social optimum for the commons corresponds to choosing the industry fleet size that maximizes total *industry* profit Π for competitive markets. The maximand is

$$(8) \qquad \Pi(R) = P \cdot F(R) - wR.$$

In maximizing this expression, P is considered parametric to the social planner. The social optimum fleet size R^* satisfies $P(R^*) \cdot F(R^*) = w$, and is independent of the distribution of catch between firms. Since demand is downwards sloping, (1) implies that the exploitation rate associated with (7) is less than the Pareto-optimal rate. Thus, the single exploiter's fleet size and catch rate for the commons are suboptimal under conditions of imperfect competition; monopoly exploitation implies too much conservation. This observation raises an interesting question concerning the fleet and catch rates, $\hat{R}$ and $\hat{C}$, that satisfy the Nash condition depicted in (5). Namely, can a Pareto optimum number of exploiters be found such that the desire to underexploit, associated with market imperfection, exactly offsets the tendency to overexploit, associated with the commons? To answer this question, the exploitation and catch rates that satisfy (5) must be equated to those rates associated with Pareto optimality, where $P \cdot F'(R^*) = w$. Letting $\hat{R} = R^*$ and $\hat{C} = C^*$, we can solve for the number of firms, where Nash exploitation also satisfies Pareto optimality. This, then, gives the social optimum number of firms, n^*, in terms of the elasticity of input productivity [i.e., $\varepsilon_C = RF'(R)/C$] and the price elasticity of market demand, ε_D:[4]

$$n^* = 1 + \varepsilon_C/[(\varepsilon_C - 1)\varepsilon_D]. \tag{9}$$

By (2), the value of the elasticity of input productivity lies between zero and one. Hence, $n^* \geqq 1$.

Equation (9) represents the social optimum number of firms in terms of the relevant demand and supply elasticities, when Nash conjectures characterize the commons. If, for example, market demand is perfectly elastic, then (9) implies the conventional wisdom: a single firm must exploit the commons to achieve a social optimum. For a given elasticity of input productivity, (9) indicates that the more inelastic is market demand, then the greater should be the number of firms in the commons if a social optimum is to be achieved. This follows because the increase in market imperfection leads to a greater degree of conservation as firms restrict output, thereby taking advantage of buyers' unresponsiveness to price. Hence, more exploiters can be admitted to the commons, since the resulting increase in exploitation is needed to offset the conservation associated with monopoly power. For imperfect competition, a single exploiter is therefore not necessarily an optimal arrangement for the commons. Holding market demand elasticity fixed allows us to focus on the influence of input-side distortions by varying the elasticity of input productivity. In (9), as ε_C nears one, n^* approaches infinity, implying that free access or unre-

4. To derive (9), replace r/R in (5) by $1/n$, and divide both sides of (5) by $P(C)F'(R)$. These operations yield

$$\frac{C}{RF'(R)} - \frac{w}{F'(R)P(C)} + \frac{1}{n}\left[1 + \frac{P'(C)C}{P(C)} - \frac{C}{RF'(R)}\right] = 0. \tag{5'}$$

Using the Pareto-optimal requirement, $w/[F'(R)P(C)] = 1$, and rewriting the expressions in (5′) in terms of the relevant elasticities yield

$$\frac{1}{\varepsilon_C} - 1 + \frac{1}{n^*}\left[1 + \frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_C}\right] = 0. \tag{5''}$$

Solving for n^* gives (9).

stricted entry is desirable. If, for example, $\varepsilon_C = 1$, then average and marginal products are equal; hence, equating average product to the real rental rate leads to no input-side distortion. As diminishing returns increase and ε_C approaches zero, the commons problem intensifies and the number of exploiters must be restricted as shown by (9).

II. Concluding Remarks

In the above analysis, we have related the social optimum number of firms to a demand-side and a supply-side elasticity when firms employ Nash, zero conjectural variations. Our results easily generalize to non-Cournot conjectures as long as an increase in the number of firms strictly increases market power. (Of course, (9) will have to be altered if conjectures are non-Cournot.) However, the results of this paper do not generalize to "consistent" conjectures [Bresnahan, 1981]. Elsewhere, we have shown that the only consistent conjectures for the commons problem are -1.[5] For the commons, a consistent conjecture drives profits to zero [Cornes and Sandler, 1983] for any number of firms. Thus, no n^* exists under consistent conjectures.

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5. The proofs of these statements are available in a longer version of this paper available from the authors on request.

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