Identifying and Evaluating Strategies for Teaching Measurement According to Student Conceptual Development

By

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Plan B Project

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Abstract

Low scores on the measurement portion of the yearly Proficiency Assessment for Wyoming Students (PAWS) Mathematics test in sixth grade at Rawlins Middle School led to the research for classroom materials that could better improve student understanding. A literature review identified developmental factors and instructional strategies for promoting student understandings of concepts and processes of measurement and of perimeter and area. These were incorporated into a unit plan to teach to a 2011-2012 sixth grade class. The instructional plans were pilot tested and modified based on student assessments and teacher observations. The unit plan, which took two weeks to complete, consisted of a pre- and post-assessment, lessons, and worksheets. The pre- and post-assessment, along with student work were used to determine whether there was an improvement in student understanding. Overall, for this group of students, there was an improvement in the level of understanding; limitations to this study are also discussed.

This paper is dedicated to my mom, Deb, who helped me reach this goal

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Chapter One

Introduction

Sixth grade students at Rawlins Middle School perform poorly on the Measurement portion, specifically the perimeter and area, of the Proficiency Assessments for Wyoming Students (PAWS) test. However, they are proficient or advanced overall on the PAWS test, though not 100%. Table 1 shows their performance from 2007-2012:

Table 1

PAWS Results from 2008-2010 (% of proficient sixth grade students)

School Year	Perimeter/Area	Overall	AYP Target
2007-2008	30.1	65.5	49.20
2008-2009	31.1	55.8	49.20
2009-2010	44.4	58.1	49.20

Note: AYP = Annual Yearly Progress

These data, along with an apparent lack of student understanding during class, as noted by the superficial understanding of the lessons taught in class, reinforced one of my concerns as a teacher. Do the mathematics resources currently in place address the conceptual development needed for student understanding? The current textbook series used at my school is McDougal Littell, *Mathematics Course I* (2007). Before it was chosen, the mathematics department met and created a type of rubric we would use to decide among the variety of texts offered to us. We were looking for a text that included material that corresponded to each of our district's five mathematics standards and benchmarks (See Appendix A). We chose this textbook series because it scored high on our rubric, included lessons that were easy to use for both teacher and student, and provided Internet resources that we found useful. The mathematics teachers at Rawlins Middle School have been using this textbook for five years beginning in 2007.

Statement of the Problem

The textbook is designed to teach sixth grade mathematics standards in a way that builds skills as they progress through the school year. Chapter one includes lessons on whole number operations, powers and exponents, order of operations, variables and expressions, and equations and mental mathematics. Chapter two begins with lessons on linear measurement, perimeter and area, graphing, and mean, median, and mode. Chapters three and four cover decimal operations and chapters five, six, and seven cover fraction operations. Concepts from chapter one, order of operations and variables and expressions, are used when working with decimals and fractions. The rest of the textbook covers ratio, proportion, and percent, geometry, integers and equations, and probability.

According to the textbook, perimeter and area are two of the first lessons that students would encounter in sixth grade. Because students are more independent, willing to solve problems on their own, and they are able to understand abstract concepts better later in the year; I wait to teach the lessons from chapter two during third quarter, after our unit on geometry, right before the Proficiency Assessments for Wyoming Students

(PAWS) testing[1]. I want perimeter and area to be fresh in the students' minds when taking the PAWS test.

The textbook teaches these concepts through the use of formulas, which are provided for perimeter and area. Together, the students and I read the instructions from the book, work out the textbook example problems on the board, and then complete the assignment that asks them to find perimeter and area using the given formulas. After I help students with the vocabulary portion of the assignment, students will begin working on their assignment, but often ask, "Which one is perimeter/area again?" or "Am I supposed to add up all the sides or multiply?" or even "Do I multiply all of the sides?" They do not refer back to the formulas given in the textbook and they do not label area with squared units and perimeter with linear units.

I have also used a supplemental activity called "The Home Design Project" from Scholastic Professional Books (2003). The majority of the lesson focuses on students being able to design their "dream house" using grid paper that uses a scale of one edge of a square = two feet. They use this to find the square footage of each room, a garage, and a pool. They also have to find the perimeter of their garden. My hope is that this lesson will give them a chance to use perimeter and area in a "real-world" setting. What I have found is that my students understand that they have to multiply the length times the width

[1] PAWS is conceptually constructed around an instructionally supportive design to include clear targets for instruction and informative reporting categories. PAWS results provide skill-level reporting categories aligned to the Wyoming Content and Performance Standards as organized by the Wyoming Assessment Descriptions to assist teachers in interpreting and addressing specific academic needs of students.

to find area but see area as only a formula. They are unsure of how to find the perimeter of the garden. Again, they are unable to label area in square units and perimeter in linear units. During the chapter test, students seem to forget what they have learned when it comes to area and perimeter. They confuse the meaning of each term and they do not know how to use the formulas to find each measurement.

I have seen the same lack of understanding of perimeter and area for the past eight years I have been teaching mathematics at Rawlins Middle School. Students consistently confuse area and perimeter formulas, they are unable to define the terms, and they often need me to help them answer questions about perimeter and area in a step-by-step fashion. This occurs during the textbook assignment, the Home Design Project, and the chapter test. It feels as though I am only introducing the basic formula when students are learning how to measure instead of gaining a deep understanding of this concept.

The activities I used in the classroom had these same results for the last eight years, which suggests that the current resources used for teaching perimeter and area have been unsuccessful. I believe the reasons for this lack of understanding may also result from factors other than the resources used in class. By doing this project, I am seeking to find more reasons for these misunderstanding through a review of the literature. In addition, I will find or create instructional materials to address these misunderstandings.

Research Questions

This study is designed to help answer the following questions:

1. What are the effective strategies to develop the conceptual understanding of sixth grade students with regard to measurement, particularly perimeter and area?

2. Can I determine if various instructional resources will address these effective strategies in my classroom?

I will address these questions by reviewing the literature on conceptual development of measurement and effective strategies used in the teaching and learning of measurement in general, and area and perimeter in particular. Additionally, I will create and test a unit plan that includes this conceptual development in the classroom.

Chapter Two

Review of the Literature

Introduction

This literature review will focus on concepts needed to develop an appropriate understanding of measurement, specifically perimeter and area. Throughout, I will provide examples for each concept and also include possible misconceptions that students may encounter when measuring. In addition, I will review different types of evaluation tools in order to create a tool to be used to evaluate resources for conceptual development strategies.

Conceptual Development for Measurement

There are several concepts students must learn in order to be able to measure correctly. The Canadian Ministry of Education (2008) states, "In the primary grades, students learn to estimate, measure, and record length, height, distance, area, capacity, and mass, using non-standard and standard units (p. 13)." However, teachers tend to focus on the skills of each of these tasks without taking into consideration the concepts that are involved. In relationship to measurement, students should develop an understanding of many different concepts associated with the skills in order to better understand measurement, specifically perimeter (linear measurement) and area.

Linear measurement. Results from the NAEP international assessments indicate that students' understanding of measurement lags behind all other mathematics topics (National Center for Education Statistics 1996). One reason for this is that teachers may be unaware of the concepts students need to learn in order to reason when measuring. Many of the skills needed in order to measure include being able to reason. Stephan and

Clements (2003) identified six concepts students must learn in order to perform linear measurement. They are: (a) partitioning, (b) unit iteration, (c) transitivity, (d) conservation, (e) accumulation of distance, and (f) relation to number. The following descriptions include reasoning activities.

Partitioning. Stephan & Clements (2003) defined partitioning as the mental activity of slicing up the length of an object into the same-sized units. This is a basic skill that may involve activities in which students create their own ruler by partitioning a strip of paper into equal pieces using hash marks. Students can do this initially by using a variety of objects such as paper clips, beans, and cubes to partition, but will eventually be expected to master the use of standard units such as inches, feet, and yards. Mastery of partitioning happens when students understand that length is continuous, or that any length can be cut into smaller pieces.

Unit Iteration. Unit iteration requires the repeated application of a unit on the object to be measured (Steffe & Hirsten, 1976). In order to do this, students must be able to choose a unit, such as a paper clip, bean, or cube, place it along the length of an object and count the number of objects needed to measure the object. There are misconceptions that may be encountered. These include leaving gaps when measuring. This means the students are focusing on the counting part of the activity instead of seeing that the unit is covering an amount of space. It is important to stress the meaning of each number as the amount of space being covered when counting so that students realize that a "four" represents the units of space being covered and not just the hash mark on a ruler.

Students may also try to use different sized units to measure one object or try to start counting at the number "one" on a ruler. This shows that students regard this as a counting activity instead of measuring an amount of space. They are taught to line up the zero on the ruler at one end of the object and refer to the number at the right of the object to find that object's length, but unless they understand why they must begin with zero, they may not really understand it at all. Levine, Kwon, Huttenlocher, Ratliff, & Deitz (2009) examined what happens when students are faced with an object that is not lined up with the zero, or misaligned, on the ruler. Often, students still refer to the number at the right to determine the length of the object. Some students look at the hash marks instead of finding the number of units needed to measure the object. If they just count them, it may result in the measurement of the object being one unit short.

In other cases, it seems that students are dependent upon a procedure for finding length instead of using unit iteration. Levine, Kwon, Huttenlocher, Ratliff, & Deitz, (2009) noted, "Rather, they seem to be gaining a set of procedural skills that make them appear to understand measurement" (p.2391). Though discrete units are used to teach unit iteration, students often are not given the opportunity to compare their results between discrete units and results from using a ruler. They are also not given opportunities to compare results between misaligned and aligned ruler measurements.

Piaget, Inhelder, and Szeminska (1960) demonstrated how children use the substitutive property (if a = b, then a can replace b in any equation) for unit iteration by solving different problems showing distance traveled on two parallel strings with beads representing trains. At approximately age seven, children could find equal distances as long as the departure points were the same, the strings were parallel, and the beads traveled in the same direction. According to Piaget, et al., (1960), children, ages eight and older, could solve the problems using departure points at opposite ends of the strings and beads that traveled in opposite directions and are said to be reasoning by using unit iteration.

Transitivity. The concept of transitivity was defined by Stephan and Clements (2003) as the understanding of the following rules: (a) if the length of object one is equal to the length of object two and object two is the same length of object three, then object one is the same length as object three; (b) if the length of object one is greater than the length of object two and object two is longer than object three, then object one is longer than object three; and (c) if the length of object one is less than the length of object two and object two is shorter than object three, then object one is shorter than object three. Research on conceptual development (Piaget, Inhelder, and Szeminska, 1960; Steffe and Hirsten, 1976) concluded that children engage in the use of the transitive property at different stages. When comparing the height of two towers, children aged four to five will make the comparison visually without moving the towers next to one another. Children from ages five to seven will use manual transfer by moving the towers close together. Children aged eight and older will use a longer tower to compare the two towers and will make statements such as “The longest tower is longer than both shorter towers.”

Conservation of Length. Conservation of length is the understanding that as an object is moved, its length does not change (Stephan & Clements, 2003). Students should be able see that two strips of paper are still the same length even if one of the strips of paper is moved. Several researchers (Piaget, Inhelder, and Szeminska, 1960) presented students with two strips of paper of the same length each in the same position. Students were able to see that the two papers were of the same length, however, when one paper was moved forward a few centimeters, students who could not conserve length no longer agreed the papers were the same length. Kidder and Lamb (1981) used both continuous and discrete objects to test for conservation of length with students from grade levels 2, 3, and 4 to see whether conservation is needed in order to understand how to measure. Researchers debate whether transitivity and conservation of length are needed to have a complete understanding of measurement. Piaget argued that without conservation of length, students could not understand transitivity because of the changing length.

Accumulation of Distance. Stephan and Clements (2003) defined the accumulation of distance as the result of iterating a unit that signifies, for students, the distance from the beginning of the first iteration to the end of the last. For example, students understand accumulation of distance when they are able to measure the distance of a room using footsteps and seeing that the "seventh" step represents the distance from the beginning of the first step to the end of the seventh step. Students first understand this concept as young as six years old, though mastery occurs around age nine (Piaget, Inhelder, and Szeminska 1960).

Relation between Number and Measurement. Measuring is related to numbers in that measuring is simply a case of counting. However, measuring is conceptually more advanced since students must reorganize their understanding of the very objects they are counting (discrete versus continuous units) (Stephan & Clements, 2003). One example of the relation between number and measurement involves an activity where students compare two equal lengths made up by different sized matchsticks. Students who struggle with this concept will say that the length made up by a greater amount of shorter matchsticks is the longer length. In this example, students are confusing their counting skills with measurement concepts.

Area. Measuring the area of an object can be very difficult for students. Students often do not have the skills needed to find the linear measurement of each side of the object, let alone understand that they need to multiply the two measurements to find the area. Students also have a hard time creating an array, or an arrangement of objects into columns and rows. This is necessary to find the area of an object. Stephan and Clements identified four concepts necessary for conceptual understanding of calculating area. These are: (a) partitioning, (b) unit iteration, (c) conservation, and (d) structuring an array.

Partitioning. Partitioning is the mental act of dividing two-dimensional space into two-dimensional units (Stephan & Clements, 2003). Students' first experiences when learning how to measure the area of an object might include tiling a two-dimensional space using a two-dimensional unit of choice, whether it is a bean or a wooden tile. Students should gain practice in partitioning a space into a number of units.

Unit Iteration. Unit iteration is the ability to "cover regions with area units" (Stephan & Clements, 2003). Students should be given the opportunity to use tiles to cover the space of an object. There should be no gaps or overlapping of units. The goal is that students begin to see the units as the structuring of an array. The use of manipulatives would be necessary for this activity.

"It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas" (Van de Walle, 2001, p. 32). Cass, Cates, Smith, and Jackson, (2003) used geoboards and guided practice to teach area and perimeter to a group of special education students. “Employment of concrete manipulatives in conjunction with modeling, guided practice, and independent practice helped students determine the correct procedures to use when computing the area and perimeter of various figures they encounter in everyday life” (Cass, Cates, Smith, & Jackson, 2003, p. 119).

“Concrete materials may conceal the very relations they are intended to illustrate” (Outhred & Mitchelmore, 2000, p. 146). Outhred and Mitchelmore (2000) argued that students are more successful when using wooden tiles to cover a surface because the use of wooden tiles prevent students from overlapping or leaving gaps which makes the task too easy. Students fail to see the structure of the array or the relation to the formula for area. “A teacher needs to be aware of multiple interpretations of materials in order to hear hints of those which students actually make. Without this awareness it is easy to presume that students see what we intend they see, and communication between teacher and student can break down when students see something other than what we presume” (Thompson, 1994, p. 557).

One example of this is the use of squared paper. Though using squares to show how to make length multiplied by width understandable, students could become confused. When measuring the length and width of an area, students use the side edge of the square, but when measuring the area of an object, students use the inside area of the square. Students may become confused when asked to count the number of squares to measure both the length and width of a figure and the number of squares inside of a figure. Also, when working with squared paper, students may have a hard time finding the number of squares an irregular shape covers.

Conservation of Area. Conservation of area requires students to accept that when they cut a given region and rearrange its parts to form another shape, the area remains the same (Stephan & Clements, 2003). Piaget (1960) found similar results when working with children between the ages of six and seven. When asked if a transformed figure had "the same amount of room" as the original figure, children of ages six to seven had no sense of conservation of area whereas children ages seven or eight could see that the transformation did not alter the amount of space. Hirstein, Lamb, and Osborne (1978) observed children who were asked to refer to an original figure to create an equal amount of area on a comparison 'strip' (the comparison strip was usually two units wide compared to a variety of widths of the original figures). Squares, rectangles, triangles, and L-shaped figures were used as original figures. One portion of the problem used grid paper, one portion used indicated grids, and another portion didn't use any grids at all. In this study, Hirstein, et al. (1978) discovered five misconceptions in the behaviors of the students. First, students used the length of one dimension to make area judgments. For example, students would focus on the length of the original figure and make sure to mark the same length on the comparison strip even though the widths of the figures did not match. Second, students used primitive compensation methods, which included justification for why they made the length longer on the comparison strip; students were attempting to make up for the shorter width on the comparison strip when comparing a square with sides of six units to the comparison strip that was only two units wide. The next misconception involved point counting. Students used this to determine the area. When they did this, they would refer to a point in the middle of the unit instead of seeing the unit as a space-occupying geometric entity. This made it difficult for students to see that a unit can be divided into halves and still joined them to make a whole. Fourth, students also counted around the corner going from the width to the length to find one total number of units. For example, students would count the number of squares for the width and continue around the corner and add the number of squares for the length and use this total for the number of units of area.

One final misconception Hirstein, Lamb, and Osborne discovered was that students point-counted linear units. Students had no sense of the linear unit. Instead, students mistook the marks at the end of the unit and counted those instead. This made them come up one short every time.

Structuring an Array. Structuring an array, or an arrangement of objects in rows and columns, develops through a series of levels found by Battista, Clements, Arnoff, Battista, & Van Auken Borrow (1998). Table 2 shows the different levels of understanding, definitions of each level and what skills students have at each level:

Table 2

Levels of Development of Structuring an Array

Level	Definition	Student Ability
Level 1	No use of a row or column of squares as a composite unit (a “line” of squares thought of as a group).	Students at this level have difficulty visualizing the location of squares in an array and counting square tiles that cover the interior of the rectangle.
Level 2	Partial row or column structuring.	Some students, for example, make two rows but no more.
Level 3A	Structuring an array as a set of row- or column-composites.	Students at this level see the rectangle as covered by copies of composite units (rows or columns) but cannot coordinate those with the other dimension.
Level 3B	Visual row- or column-iteration.	These student can iterate a row (e.g., count by fours) if they can see those rows.
Level 3C	Interiorized row- or column-iteration.	These students can iterate a row using the number of squares in a column; for example, five rows of four square units equals 20 square units. Only at this level is the usual “formula” method of determining area going to have a firm conceptual basis for most students.

Outhred and Mitchelmore (2000) identified levels of understanding of the structure of arrays. Students were asked to determine the area of an eight by ten rectangle using three different processes. In one process, students were allowed to use a moveable tile to cover the shape and find the area. In yet another process, students could refer to an immovable unit to determine the area. Finally, students were asked to find the area of the rectangle when neither the unit nor the rectangle was shown. Outhred and Mitchelmore (2000) discovered that students work at five different levels of understanding: (a) incomplete covering, including gaps and overlays; (b) primitive covering (unsystematic covering); (c) array covering, constructed from units; (d) array covering, constructed from measurement; and (e) array implied with a solution by calculator.

When working with the area formula, Kai Kow Joseph Yeo (2008) argues that it is extremely difficult for pupils to understand how two lines (the length and the width) can produce an area when they are multiplied.

Perimeter and Area

One problem students encounter when working with perimeter and area is that they are often taught to memorize the formulas for perimeter and area by rote and are never asked to (*or* given the opportunity to) investigate how they were created. This means they do not have a clear understanding of why the formulas work. Muir (2006) stated, "Introducing the area formula before students have had opportunities to develop a conceptual understanding of area and to see the usefulness of arrays could be counter-productive to developing sound measurement sense" (p. 8).

Vighi and Marchini (2011) identified the following "conflicts" of perimeter and area. The first conflict is students often believe that everything is measurable with a ruler, which leads to confusion when finding area. Another conflict is "false conservation", in which students assume that if the perimeter of a figure increases, so does the area. This occurs for area versus perimeter, as well.

After reviewing the literature, I have identified several concepts needed when learning how to measure both perimeter and area. Also included are strategies for teaching these concepts and misconceptions students may encounter when measuring.

Lesson Evaluation

The low performance in the content area of mathematics by students in the United States, according to results from the NAEP international assessments, leads to the question of what can be done to improve mathematics proficiency (National Center for Education Statistics 1996). Often there is a focus on classroom resources. Though there are many factors that affect a students' understanding of mathematics, the type of curriculum materials used in the United States' educational systems has been researched extensively.

A research study done by Clark-Wilson (2008) looked at how the implementation of technology impacted the teaching approaches and learning outcomes of students in secondary mathematics classrooms. She found that the majority of her students were able to use the technology to engage with the tasks. She also discovered the importance of asking certain questions in order to evaluate the use of a new educational resource, in this case the TI-Nspire, in the classroom. A few of the questions include: "What were students' initial reactions/questions?" "What aspect(s) of the resource would you use again?" and "What changes would you make?"

Iguchi and Suzuki (1996) used Keller's (1992) ARCS motivational design factor when they incorporated Cabri Geometry into a 9^{th} grade geometry class. ARCS looks at four factors: Attention, Relevance, Confidence, and Satisfaction. They also administered a term beginning, midterm, and final examination. They used Cabri Geometry in one class while keeping the other three classes control groups. Though students had an improved Confidence from the use of Cabri Geometry, their test scores were the same as the control group scores.

For my study, I incorporated the use of the pre- and post-assessments and observations to evaluate the effectiveness of the unit plan.

Chapter Three

Methods

Goals

For the last eight years teaching sixth grade mathematics at Rawlins Middle School, I have noticed that the Proficiency Assessments for Wyoming Students (PAWS) results show that students are not as proficient at solving perimeter and area problems as they are in other areas. The goal for this study was to identify effective strategies that could help sixth grade students to develop a conceptual understanding of the measurement topics of perimeter and area. I also tested various lessons and materials that used these effective strategies. To do this, I borrowed, altered, and created lessons that utilized these strategies and implemented them in my classroom.

Classroom Setting

The school district I teach in is located in a gas and oil production community in a sparsely populated western state. A majority of my students' families depend on the oil or gas industries for employment. The student population is comprised of a majority of white students (67.7%), a significant Hispanic population (30.6%), and a smattering of other ethnic groups including Native Americans (0.8%), Asian/Pacific Islanders (0.6%), and African Americans (0.3%). The percent of students who qualify for free or reduced lunch is 38.0%, and 12.7% of the students have Individualized Education Plans (IEPs).

I initially tested the materials on a pilot group made up of 13 mathematics "intervention" students. The Mathematics Intervention classes at my middle school were created to implement a supplemental mathematics resource, Number Worlds, to help the students become proficient in different mathematics standards. The instructional facilitator and I identified students with the lowest RIT scores (for Rasch Unit) on the mathematics portion of the MAPS test. These students were placed in the mathematics intervention class for the third quarter of the 2011-2012 school year. Table 3 shows the students according to the reason for their placement in the mathematics classes.

Table 3

A Summary of Student Groups According to MAPS and IEP data

Period	Low MAPS Scores	Mathematic IEP Students
2	12 out of 17	2
7*	12 out of 13	0

Note: *Intervention

After the intervention class finished their unit on "Fractions, Decimals, and Percents" in *Number Worlds*, I taught my unit plan on perimeter and area. I wanted to pilot my unit plan with the intervention class before I used it in my sample to check for any changes that needed to be made. The sample was made up of a class of "low" level mathematics students who scored with 40% or less proficiency on the Mathematics portion of the Spring MAPS testing in fifth grade. Students who had a disability in Mathematics and were on an IEP were also placed in the low-level mathematics class. There was a Special Education Para-professional in the "low" level mathematics class who provided more one-on-one help for the students. Table 4 shows the actual population numbers for the intervention class and the sample group:

Table 4

A Summary of Math Intervention and Sample Groups

Period	Number of Students	Boys	Girls
2	21	10	11
7*	13	10	3

Note: *Intervention

Getting Started

The first step in this project was to create a unit plan that included a pre- and post-assessment and lesson plans that addressed the concepts and reasoning strategies discovered in the literature review of my project. The pre-assessment checked to see if students could: (a) identify situations where perimeter or area are used, (b) find the perimeter and area of squares and rectangles that are filled with squared centimeters and those that are not, and (c) find the perimeter and area of a figure described by words. The post-assessment was designed to be like the pre-assessment with parallel questions that were the same conceptually, but included different dimensions. Both pre- and post-assessments parallel questions were reviewed and accepted by educational professionals and can be found in Appendix C.

After giving the pre-assessment, I implemented the lesson plans from my unit plan. I chose lesson plans that addressed the concepts needed as a base for understanding perimeter. These lessons also provided an opportunity for student reasoning. These concepts were: (a) partitioning, (b) unit iteration, (c) transitivity, (d) conservation, (e) accumulation of distance, and (f) relation to number; and area: (a) partitioning, (b) unit iteration, (c) conservation, and (d) structuring an array. I chose some of the plans from my classroom materials and others came from Dr. Larry Hatfield, a professor at the University of Wyoming. The entire unit plan was sent to the educational experts on my committee for editing. The lesson plans can be found in Appendix B.

I used my mathematics intervention class as a “pilot group” to try out the lessons and make any necessary changes. I let the students know that I was doing this project due to my concern of the lack of student understanding of perimeter and area and as a way to complete my Master’s degree. I also informed the students that their names would not be used in my paper. Students were told that the lessons would not affect their grade so that they would not fear being incorrect.

Piloting the Assessments and Lessons

On the first day of the pilot, I handed out the pre-assessment. I reminded them that it would not affect their grade and that they needed to be honest. They were allowed to write, “I don’t know,” because I needed to know what the students did and did not know about perimeter and area at the beginning of the unit plan. During the pre-assessment, students asked me questions such as, “What is area?” and when referring to perimeter questions, “Do we add the sides?” and “Isn’t perimeter side times side?” This confirmed my suspicions that my students were confused about working with perimeter and area.

After reviewing the pilot group’s pre-assessment results, I was concerned whether there were enough questions that addressed the identification of perimeter and area. I did not feel that students’ answers to the two true or false questions that addressed the identification of perimeter and area informed me of whether they could identify both measurements. So for the pre-assessment used in the formal group, I changed that section to include six questions that asked students to identify perimeter and area. For example, students were given a problem where they had to find the amount of carpet for a bedroom. They needed to decide if they were finding the perimeter or area of the bedroom. Other examples can be found in Appendix B.

I was also concerned whether the pre- and post-assessments were showing a procedural knowledge of finding perimeter and area. I wanted the students to be able to use what they know about perimeter and area to create a shape with certain dimensions instead of just using the formulae for perimeter and area. With the help of my team, I added a part to the pre- and post-assessments for the formal group that would address their procedural understanding of perimeter and area. I included an activity that asked students to create a shape that had a certain perimeter and area. The post-assessment was given as the last activity of the unit plan. Before the students started, we discussed which lessons helped them remember what perimeter and area were. We also reviewed the labels that needed to be used for each measurement. The pre- and post-assessments along with their sub categories can be found in Appendix B.

Table 5 lists the daily lessons used in class along with a brief description of each:

Table 5

Unit Plan Lessons

Day	Title	Description
One	Giant Steps	Lesson that uses different non-standard units to measure the length of a classroom
	Mother, May I?	Supplemental game that incorporates the use of non-standard units
	Logo	Supplemental computer activity that incorporates standard units
Two	Cover the Desk	Lesson that uses both standard and non-standard units to cover a desk
	Area and Perimeter Tiles	Lesson that has students represent the perimeter and area of a tiled shape by standing next to the edge of a tile perimeter or inside the tile for area
Three	Pentominoes and Pattern Blocks	Lessons that work with a fixed area and changing perimeter
Four	Puppy Pen	Lesson that works with a fixed perimeter and changing area on a centimeter grid paper
	Fenced In	Lesson that works with a fixed perimeter and changing perimeter without grids on a paper

The lesson plans used in the pilot study went very well. There were very few changes made. The first change was to the lesson that asked students to cover their desk with paper. Students were asked to use their hands to cover their desks first in order to incorporate the use of non-standard units. I made this change to reinforce that fact that area involves covering space. The other change involved the lesson with the rancher and the amount of area created with a set amount of fencing. I changed it so that students were not allowed to use the centimeter grid paper. I wanted students to be able work with perimeter and area without the use of squared units since there will be situations where they may have a figure with a length of forty feet and a width of thirty feet where they need to be able to visualize these dimensions instead of using squared units to represent them.

Data Collection

The sample included data from my 2nd hour mathematics class. Table 6 shows the different types of data that were collected for evaluation.

Table 6

Unit Plan Evaluation Methods

Day	Title	Data Collected
One	Giant Steps	Worksheet
	Mother, May I?	Observations
	Logo	Observations
Two	Cover the Desk	Observations
	Area and Perimeter Tiles	Worksheet
Three	Pentominoes and Pattern Blocks	Worksheet
Four	Puppy Pen	Graph paper
	Fenced In	Blank paper

Data Analysis

For this study, I used the pre- and post-assessments as the main indicator to determine if the lessons in my unit plan were effective. I separated each assessment into the following sections: (a) perimeter and area identification, (b) perimeter part one (figures with grids), (c) area part one (figures with grids), (d) perimeter part two (figures without grids), (e) area part two (figures without grids), (f) word problems, and (g) perimeter/area relationships.

Worksheets were collected and looked at to check for student understanding of that day's concept. Because all of the lessons were done as a class, or with a partner, a quick check for participation was all that was needed. One other piece of data that was analyzed was student responses to questions asked out loud and general comments that students made. I was looking for misconceptions that students held at the beginning of the unit and "aha" moments throughout the unit.

Chapter Four

Results

Review

As you may recall, the students that participated in this unit plan were those that showed a lack of proficiency according to previous MAPS testing. The findings may not apply to other sixth grade classes, but the unit plan used in this classroom formed a better understanding of measurement, specifically perimeter and area for students. Though there were areas that did not show a great improvement, overall students scored better on the post-assessment than the pre-assessment. The following shows the results from the activity worksheets and assessments.

Activity Worksheets

Worksheets were collected for each lesson and checked for correct completion. Lessons that required participation only were observed. Appendix B shows the different lessons in detail along with the materials that were collected as data. Table 7 shows the results.

Table 7

Unit Plan Activities

Day	Title	Data Collected
One	"Giant Steps"	All worksheets completed correctly
	"Mother, May I?"	All students participated
	"Logo"	All students participated
Two	"Cover the Desk"	All students participated
	"Area and Perimeter Tiles"	12/17 students completed the worksheet correctly
Three	"Pentominoes and Pattern Blocks"	17/17 students completed the worksheet activity correctly
Four	"Puppy Pen"	All students completed the worksheet correctly
	"Fenced In"	All students completed the worksheet correctly

The lesson worksheets showed student understanding along with appropriate participation. Students were able to define both perimeter and area when asked out loud. Students were eager to share their results especially when finding the smallest or largest perimeter and area.

Pre- and Post-Assessment Comparison

Students scored at different levels in both the pre- and post-assessments. Most of the sections on the assessment were made up of either three or six questions, so I broke up the percentages into common levels. Table 8 and Table 9 show the number of students who scored at different levels of percentage.

Table 8

A Comparison of Pre-Assessment Results for 2nd Hour

Section	Number of students with 0% to 33%	Number of students with 34% to 66%	Number of students with 67% to 100%
Perimeter and Area Identification	8	5	4
Perimeter/Part One	9	2	6
Area/Part One	11	3	3
Perimeter/Part Two	9	3	5
Area/Part Two	13	1	3
Word Problems	10	6	1
Perimeter and Area Relationships	16	0	1

For the pre-assessment, the majority of students scored between 0% to 33%.

Table 9

A Comparison of Post-Assessment Results for 2nd Hour

Section	Number of students with 0% to33 %	Number of students with 34% to 66%	Number of students with 67% to 100%
Perimeter and Area Identification	3	3	11
Perimeter/Part One	6	2	9
Area/Part One	2	2	13
Perimeter/Part Two	6	2	9
Area/Part Two	7	2	8
Word Problems	10	3	4
Perimeter and Area Relationships	16	0	1

For the post-assessment, the majority of students scored anywhere from 67% to 100% (except for Work Problems and Perimeter and Area Relationships, which will be discussed later). Tables 8 and 9 show data by percentage scores. This Item by Student data can be found in Appendix C.

Table 10 shows the overall percentage of proficient students, along with the percent of increase between the two assessments.

Table 10

A Comparison of Pre- and Post-Assessment Results for 2nd Hour

Section	% Proficient students on pre-assessment	% Proficient students on post-assessment	Differences in pre- and post-assessments
Perimeter and Area Identification	24%	59%	+35%
Perimeter/Part One	35%	53%	+18%
Area/Part One	18%	76%	+58%
Perimeter/Part Two	29%	59%	+30%
Area/Part Two	18%	47%	+29%
Word Problems	6%	24%	+18%
Perimeter and Area Relationships	6%	6%	+0%

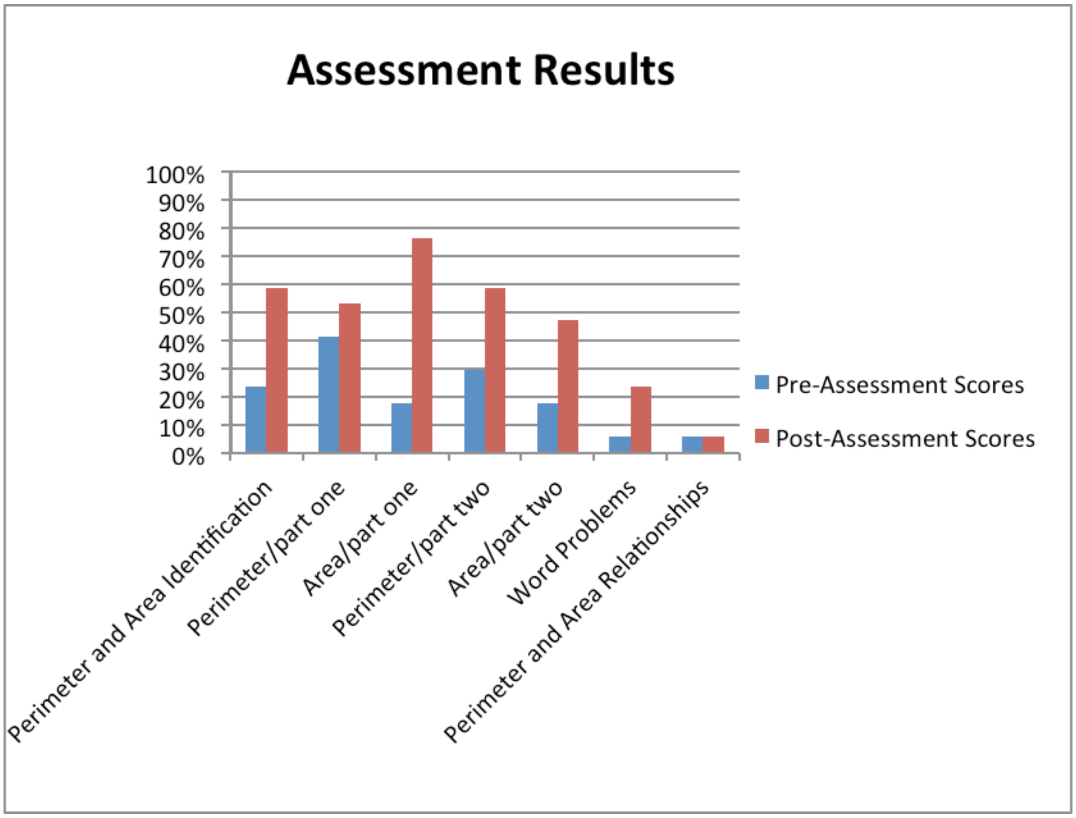

Figure 1. A Comparison of Pre- and Post-Assessment Results.

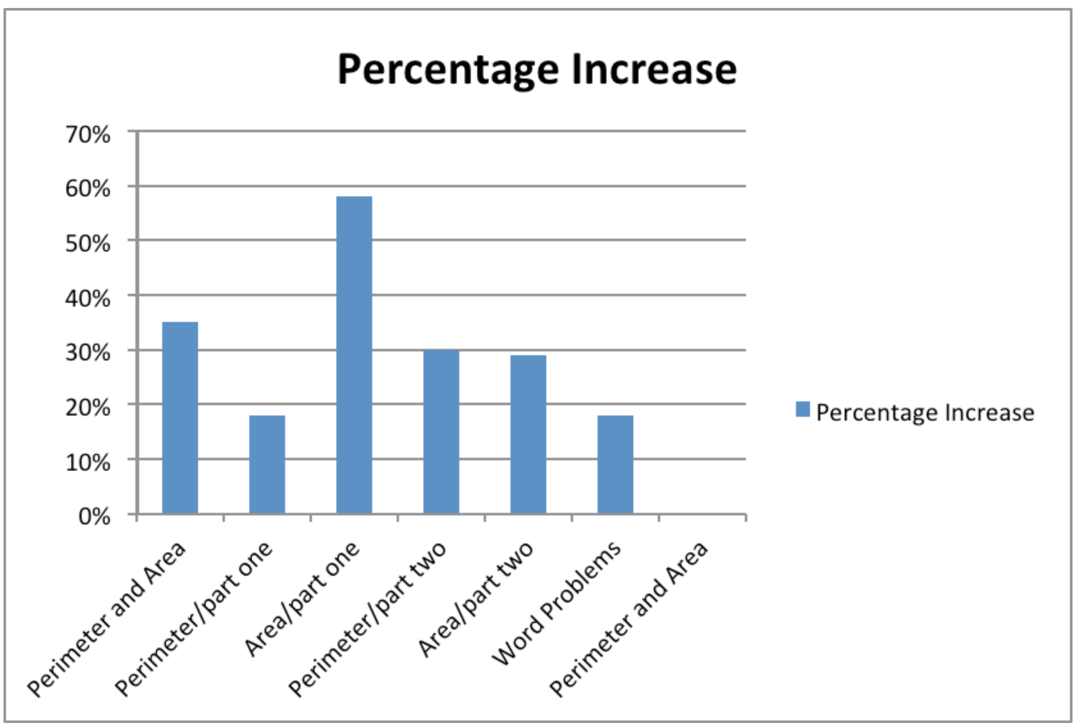

Figure 2. Percentage Increase From Pre-Assessment to Post-Assessment.

As displayed in Figure 1, students improved in all areas that were assessed except for Perimeter and Area Relationships. Some sections showed more improvement than others such as Perimeter and Area Identification (+35%), Area Part One (+58%), Perimeter Part Two (+30%), and Area Part Two (+29%). Students showed no improvement when working with Perimeter and Area Relationships.

I believe one reason for this is the fact that while working with the lessons that addressed this concept, students were not allowed to own their understanding. Instead of letting students discover the relationships between perimeter and area, I was reminding them throughout the lesson of what the relationships were.

The Item by Student data in Appendix D shows individual student scores. One set of scores that stood out were the results to the word problem section. Only one student in both the pre- and the post-assessment answered question four correctly. Something was wrong with this question for so many students to get it wrong.

One other observation made from the Item by Student data was that very few students missed both the perimeter and area questions in the post-assessment. They missed either the perimeter or area sections, but not both.

Chapter Five

Conclusions/Implications

This study suggests that incorporating a unit plan that addresses the concepts needed to learn how to measure perimeter and area may increase student understanding of measurement. Overall, students scored better on the post-assessment than they did the pre-assessment. Students also demonstrated understanding through their work with the lessons used in class. Although their scores improved from the pre- to the post-assessment, there were certain areas that were not as effective such as the Word Problems and Perimeter and Area Relationships.

Implications

One implication of this study is that through research a unit plan should be tested for effectiveness when teaching measurement. In the past I have relied on my school to supply the materials needed to teach my students. I have trusted that the materials are based on research and that they cover the required concepts. This study has shown that I need to take a closer look and be more involved in my curriculum.

Through this study, I noticed that students enjoy learning through participation. There were many comments of how much fun they were having and how they could remember what perimeter and area were by representing them using their body. There are many activities that can be incorporated into the classroom that I could be using instead of direct instruction.

Finally, I believe a teacher’s sense of worth can be improved by participating in some type of research project. I am more confident in what I am teaching to my students and I am able to share this with my teaching community along with parents and citizens in Rawlins.

Limitations

One limitation to this study would be the fact that I was unable to use a control group. It would have been interesting to see the difference between teaching from the textbook and teaching the unit plan. The reason I was unable to do this was that I wanted all students to gain an understanding of measurement from the implementation of the unit plan.

Another limitation would be that I know students' post-assessment scores improved, but I am left feeling that my unit plan was not as effective as anticipated. Student understanding of word problems along with the relationship between perimeter and area did not improve. After looking at the data from Appendix C, I noticed that question four from the word problems section had only one correct student response. Also, the perimeter and area relationship section showed no improvement at all. These may be two areas that need to be changed in the unit plan. Question four needs to be checked for rigor and I need to make sure to let students own their understanding when working with perimeter and area relationships.

Yet another limitation would be that the unit plan did not address the word problem section of the assessments. I assumed that because students were gaining an understanding of perimeter and area they would be able to answer the word problems associated with measurement. Though there was an increase in the number of students who answered the questions correctly (not including question four) the increase was not noticeable.

One last limitation was the fact that there were four students who participated in the mathematics intervention course who were also in my second hour class. Their data was not included in 2nd hour's data but I took a look at it anyway and found the following results displayed in Table 11.

Table 11

A Comparison of Pre- and Post-Assessment Results for Non-recorded Student Data

Section	% Proficient students on pre-assessment	% Proficient students on post-assessment	Differences in pre- and post-assessments
Perimeter and Area Identification	0%	50%	+50%
Perimeter/Part One	75%	50%	-25%
Area/Part One	100%	100%	+0%
Perimeter/Part Two	50%	75%	+25%
Area/Part Two	50%	50%	+0%
Word Problems	0%	25%	+25%
Perimeter and Area Relationships	0%	50%	+50%

These results are similar to the results of those students were taught this unit plan one time; however these students seemed to have a better score on the pre-assessment along with the post-assessment. Again, there were only four students in this group, so the percentage of increase may seem large, however a 50% increase means two students improved.

Due to these limitations and the fact that this is a unique group of students, my results and conclusions cannot be representative of all middle school students.

Future Research and Classroom Practices

Because of this research study, I am confident in my ability to continue to make changes to my curriculum. In our district, we are currently creating and aligning a curriculum to the Common Core Standards that will most likely be adopted in the state of Wyoming. I feel like a valued member of this process.

I have realized that I need to make changes in the way I teach, especially when it comes to dispossessed generalizations. I have a bad habit of "telling" students instead of letting students learn on their own. Students need to have more ownership in their learning.

One question I have after completing this research is whether the students' PAWS results will show an improvement in measurement, specifically perimeter and area. If not, I will continue to make changes to the unit plan. If so, I will look at the data to see which topic I would like to focus on next.

I will also share this research with the 7^{th} and 8^{th} grade teachers in my building and ask if they see a change in student understanding of measurement in the next couple of years. Because of student interest, I will continue to look for activities that get students up and moving and participating in the learning. I have always wanted my classroom to look like this, but have never followed through. This gives me the motivation to do so.

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APPENDIX A

GRADE 6 MATHEMATICS STANDARDS

CONTENT STANDARD

1. NUMBER OPERATIONS AND CONCEPTS

Students use numbers, number sense, and number relationships in a problem-solving situation.

NOTE: Students communicate the reasoning used in solving these problems. They may use tools/technology to support learning.

CODE	GRADE 6 BENCHMARKS
MA6.1.1	Students use the concept of place value to read and write decimals (to 1000ths) in words, standard, and expanded form.
MA6.1.2	Students multiply decimals (10ths & 100ths) and divide whole numbers by 2-digit divisors and divide decimals by whole numbers.
MA6.1.3	Students represent the number line using integers.
MA6.1.4	Students explain their choice of estimation and problem solving strategies and justify results when performing number operations with fractions and decimals in problem-solving situations.
MA6.1.5	Students identify prime and composite numbers and apply prime factorization to numbers less than 100.
MA6.1.6	Students demonstrate an understanding of fractions and decimals by: representing fractions as division of whole numbers; converting between mixed numbers and improper fractions; simplifying fractions and mixed numbers; writing fractions in equivalent forms; using parts of a set; rounding decimal numbers to 10ths, 100ths, and whole numbers (units) place; and converting between decimals (from .01 to .99), fractions and representing percentages.
MA6.1.7	Students add and subtract mixed numbers with like denominators.
MA6.1.8	Students represent repeated multiplication in exponential form.

GRADE 6 PERFORMANCE LEVEL DESCRIPTORS

1. NUMBER OPERATIONS AND CONCEPTS

ADVANCED PERFORMANCE

6th grade students performing at an advanced level make complex connections using number sense, place value, and estimation. They demonstrate computational fluency regardless of number size. Students use coherent and clear mathematical language to justify reasoning in problem-solving situations.

PROFICIENT PERFORMANCE

6th grade students performing at a proficient level make relevant connections using numbers, number sense, and estimation. They demonstrate computational fluency with minor errors. Students use mathematical language to communicate sound reasoning in problem-solving situations.

BASIC PERFORMANCE

6th grade students performing at a basic level make simple connections using number sense, place value, and estimation. They demonstrate limited computational skills. Students use minimal or incorrect mathematical language to communicate their thinking in problem-solving situations.

BELOW BASIC PERFORMANCE

6th grade students performing at a below basic level require extensive support or provide little or no evidence in meeting the standard.

CONTENT STANDARD

2. GEOMETRY

Students apply geometric concepts, properties, and relationships in a problem-solving situation.

NOTE: Students communicate the reasoning used in solving these problems. They may use tools/technology to support learning.

CODE	GRADE 6 BENCHMARKS
MA6.2.1	Students classify, describe, compare, and draw representations of 1- and 2- dimensional objects and angles.
MA6.2.2	Students identify and classify congruent objects by properties appropriate to grade level.
MA6.2.3	Students communicate the reasoning used in identifying geometric relationships in problem-solving situations appropriate to grade level.

GRADE 6 PERFORMANCE LEVEL DESCRIPTORS

2. GEOMETRY

ADVANCED PERFORMANCE

6th grade students performing at an advanced level make complex connections with geometric objects and attributes with or without using tools/technology. Students identify, classify, describe, and compare geometric objects using coherent and clear mathematical language. They justify problem-solving methods with valid and convincing evidence.

PROFICIENT PERFORMANCE

6th grade students performing at a proficient level make relevant connections with geometric objects and attributes with or without using tools/technology. Students identify, classify, describe, and compare geometric objects using mathematical language with minimal errors. They communicate problem-solving methods with sound reasoning.

BASIC PERFORMANCE

6th grade students performing at a basic level make simple connections with geometric objects and attributes with or without using tools/technology. Students identify and describe geometric objects using minimal or incorrect mathematical language. They communicate their problem-solving methods with limited success.

BELOW BASIC PERFORMANCE

6th grade students performing at a below basic level require extensive support or provide little or no evidence in meeting the standard.

CONTENT STANDARD

3. MEASUREMENT

Students use a variety of tools and techniques of measurement in a problem-solving situation.

NOTE: Students communicate the reasoning used in solving these problems. They may use tools/technology to support learning.

CODE	GRADE 6 BENCHMARKS
MA6.3.1	Students apply estimation and measurement of length to content problems and express the results in metric units (centimeters and meters).
MA6.3.2	Students apply estimation and measurement of weight to content problems and express the results in U.S. customary units (ounces, pounds, and tons).
MA6.3.3	Students apply estimation and measurement of capacity to content problems and express the results in U.S. customary units (teaspoons, tablespoons, cups, pints, quarts, gallons).
MA6.3.4	Students demonstrate relationships within the U.S. customary units for weight and capacity and within the metric system (centimeters to meters) in problem-solving situations.
MA6.3.5	Students determine the area and perimeter of regular polygons and the area of parallelograms, with and without models.

GRADE 6 PERFORMANCE LEVEL DESCRIPTORS

3. MEASUREMENT

ADVANCED PERFORMANCE

6th grade students performing at an advanced level make complex connections among measurement concepts accurately. They estimate, measure, and calculate using a variety of tools. Students provide valid and convincing evidence when determining area and perimeter. They use coherent and clear mathematical language to justify reasoning in problem-solving situations.

PROFICIENT PERFORMANCE

6th grade students performing at a proficient level make relevant connections among measurement concepts with minor errors. They estimate and measure using a variety of tools with and without models. Students use mathematical language to communicate sound reasoning in problem- solving situations.

BASIC PERFORMANCE

6th grade students performing at a basic level make simple connections among measurement concepts. They inconsistently estimate and measure using a variety of tools with models. Students use minimal or incorrect mathematical language to communicate their thinking in problem-solving situations.

BELOW BASIC PERFORMANCE

6th grade students performing at a below basic level require extensive support or provide little or no evidence in meeting the standard.

CONTENT STANDARD

4. ALGEBRA

Students use algebraic methods to investigate, model, and interpret patterns and functions involving numbers, shapes, data, and graphs in a problem-solving situation.

NOTE: Students communicate the reasoning used in solving these problems. They may use tools/technology to support learning.

CODE	GRADE 6 BENCHMARKS

MA6.4.1	Students recognize, describe, extend, create, and generalize patterns, such as numeric sequences, by using manipulatives, numbers, graphic representations, including charts and graphs.
MA6.4.2	Students apply their knowledge of patterns to describe a constant rate of change when solving problems.
MA6.4.3	Students represent the idea of a variable as an unknown quantity, a letter, or a symbol within any whole number operation.

GRADE 6 PERFORMANCE LEVEL DESCRIPTORS

4. ALGEBRA

ADVANCED PERFORMANCE

6th grade students performing at an advanced level make complex connections among algebraic concepts accurately. They apply and describe patterns in a problem-solving situation accurately. Students use coherent and clear mathematical language to justify reasoning in problem-solving situations.

PROFICIENT PERFORMANCE

6th grade students performing at a proficient level make relevant connections among algebraic concepts with minor errors. They apply and describe patterns in a problem-solving situation with minor errors. Students use mathematical language to communicate sound reasoning in problem-solving situations.

BASIC PERFORMANCE

6th grade students performing at a basic level make simple connections among algebraic concepts. They sometimes apply and describe patterns in a problem-solving situation with errors. Students use minimal or incorrect mathematical language to communicate their thinking in problem-solving situations.

BELOW BASIC PERFORMANCE

6th grade students performing at a below basic level require extensive support or provide little or no evidence in meeting the standard.

CONTENT STANDARD

5. DATA ANALYSIS AND PROBABILITY

Students use data analysis and probability to analyze given situations and the results of experiments.

NOTE: Students communicate the reasoning used in solving these problems. They may use tools/technology to support learning.

CODE	GRADE 6 BENCHMARKS
MA6.5.1	Students systematically collect, organize, and describe/represent numeric data using line graphs.
MA6.5.2	Students, given a scenario, recognize and communicate the likelihood of events using concepts from probability (i.e., impossible, equally likely, certain) appropriate to grade level.

GRADE 6 PERFORMANCE LEVEL DESCRIPTORS

5. DATA ANALYSIS AND PROBABILITY

ADVANCED PERFORMANCE

6th grade students performing at an advanced level make complex connections about data and probability. They collect, organize, and represent information, describe, interpret and defend results in data and probability experiments accurately. Students predict reasonable outcomes using concepts from probability. Students use coherent and clear mathematical language to justify reasoning in problem-solving situations.

PROFICIENT PERFORMANCE

6th grade students performing at a proficient level make relevant connections about data and probability. They collect, organize, and represent information, describe and interpret results in data and probability experiments with minor errors. Students predict reasonable outcomes using concepts from probability. They use mathematical language to communicate sound reasoning in problem-solving situations.

BASIC PERFORMANCE

6th grade students performing at a basic level make simple connections about data and probability. They collect, represent, and report information in data and probability experiments. They predict outcomes using concepts from probability with limited success. Students use minimal or incorrect mathematical language to communicate their thinking in problem-solving situations.

BELOW BASIC PERFORMANCE

6th grade students performing at a below basic level require extensive support or provide little or no evidence in meeting the standard.

APPENDIX B

LESSON PLANS

Student Lesson: "Giant Steps"

Description: Students use different sized steps to measure the length of a classroom.

Learner Outcomes: Students will be introduced to linear measurement through activities that address the following concepts:

- Partitioning
- Unit iteration
- Accumulation of distance

Materials:

- Worksheet to record results of steps (Attached)

Estimated time needed for lesson: 50-60 minute period

Part One

Students will use different sized units such as giant steps and baby steps, to estimate and measure the width of the classroom.

First, select a student to be the giant. Have the giant start at one wall of the room and take two giant steps toward the other wall and freeze. Ask the students to estimate the number of giant steps it would take to reach the other wall.

Next, write the estimates on the board; then ask the giant to pace three more giant steps, and record any revision of the students' estimates. Have the students count out loud the number of paces needed to reach the other wall.

Third, try the same activity using baby steps, or one foot directly in front of the other. Students should discuss how many steps they think it will take. Students need time to discuss the inverse relationship that is occurring here – the smaller the pace, the bigger the number of paces. Finally, ask students whether they would get the same number if they paced across the room. Allow for time for discussion.

Part Two

Students will work in pairs to make estimates of how many steps it would take to cross the room using three different types of steps (giant steps, baby steps, or hops). Students then find the actual number of each type of step they are using. While one student paces, the other students counts. Students then switch roles and record their results on their worksheet.

Part Three

There are two extension activities that can be used in this lesson. One is a game called "Mother May I?" This game requires around ten students along with a *caller*, who decides whether or not a player can move the proposed spaces. The caller stands on an imaginary line facing the players about 15 to 20 feet away. A player proposes a move such as "May I move five giant steps forward?" and the caller can respond, "Yes, you may," or "No, but you can take five baby steps." The first player to reach the caller takes the caller's place.

Another extension is the Logo computer activity. Students gain experience giving and adjusting directions to move the Logo turtle around the screen using the same sized unit. The National Library of Virtual Manipulatives activities Ladybug Leaf and Ladybug Mazes are other computer activities that are similar to Logo.

Giant Steps Worksheet

PACING IN PAIRS

Make an estimate of the number of paces to your target.
Then count how many paces the pacer actually takes.

Caller's name: ____________________

Pacer's name: ____________________

Estimate: __________ Estimate: __________

Actual paces: __________ Actual paces: __________

Estimate: __________ Estimate: __________

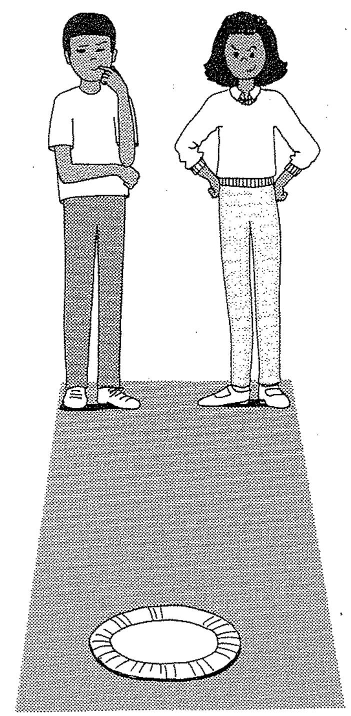
Actual paces: __________ Actual paces: __________

Estimate: __________ Estimate: __________

Actual paces: __________ Actual paces: __________

Estimate: __________ Estimate: __________

Actual paces: __________ Actual paces: __________



Student Lesson: “Cover the Desk”

Description: Students use pieces of paper to find the area of the top of their desk.

Learner Outcomes: Students will be introduced to area through activities that address the following concepts:

- Partitioning
- Unit iteration

Materials:

- Paper cut into four inch by four inch pieces

Estimated time needed for lesson: 20-30 minute period

Students are asked to look at the top of their desk and figure out how many hands they would need to cover it. Students then use their hands to cover their desk and share their results with the class. The teacher asks, “Why did we use a different number of hands to cover the desks?” The following reasons should come up: size of hands, gaps, overlays, and methods.

Students then use four inch by four inch pieces of paper to cover their desks and share their results. The teacher asks, “Why did we use similar numbers of pieces of paper to cover the desk?” The students need to recognize the standard unit being used. The teacher also needs to ask, “Why didn’t we all get the exact same answer?” The following reasons should come up: gaps, overlays, and methods.

Student Lesson: “Perimeter and Area Tiles”

Description: Students role-play “perimeter” and “area” of actual tile figures.

Learner Outcomes: Students will be introduced to linear measurement and area through activities that address the following concepts:

Linear Measurement

- Partitioning
- Unit iteration
- Accumulation of distance

Area

- Partitioning
- Unit iteration

Materials:

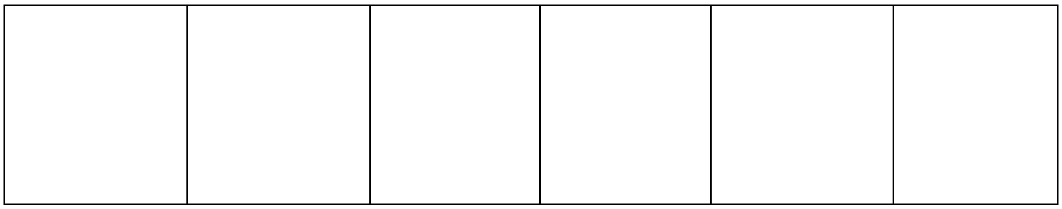
- Worksheet to record results of perimeter and area of tiles (Attached)
- Ceramic floor tiles (or one foot by one foot pieces of tag board)

Estimated time needed for lesson: 30-40 minute period

A tiled floor is needed for this activity. First, the teacher marks off several different rectangles and squares using tape and the tiles on the floor. Students are given a note card with the letter “P” or the letter “A” on it. Students with the letter “P” are asked to stand on the outside edge of the rectangle or square and students with the letter “A” are asked to stand on an inside square. The teacher emphasizes that a “P” is the outside edge and the “A” is the area covered within the square. Students then label the same rectangles and squares on their worksheet with the names of the students that represented the P's and the A's. They also determine the perimeter and area of the shapes on the worksheet. Students can also be given a "tile" to use to measure the perimeter and area of different rectangles and squares marked out on the floor.

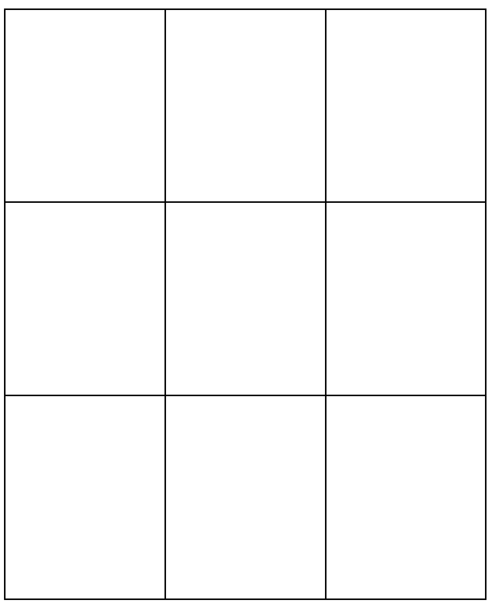
Perimeter and Area Worksheet

Please label the shapes with the person who represented the perimeter and the person that represented the area:



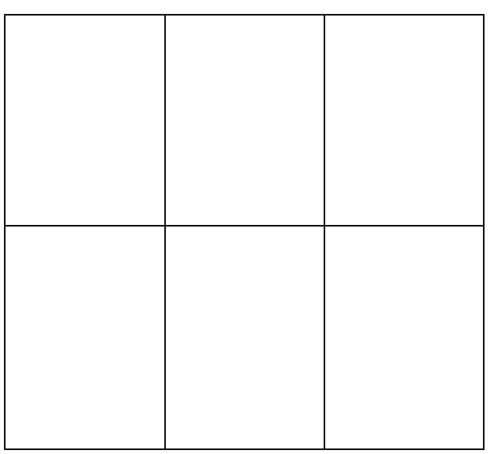
Perimeter = ______ units

Area = _____ squared units



Perimeter = _____ units

Area = _____ squared units



Perimeter = _____ units

Area = _____squared units

Student Lesson: "Pentominoes and Pattern Blocks"

Description: Students use pentominoes and pattern blocks to look for any relationship between perimeter and area.

Learner Outcomes: Students will look for relationships between perimeter and area through activities that address the following concepts:

Perimeter

- Transitivity
- Conservation of Length

Area

- Conservation
- Structuring of an Array

Materials:

- Inch graph paper
- Pattern blocks
- Pentominoes
- Worksheet (Attached)

Estimated time needed for lesson: 50-60 minute period

Students will work with a set of pentominoes, which are made up of five square inch units arranged in different patterns. They will count the number of square inches in each pentomino then find the perimeter of each pentomino. Students will see that even though the area of each pentomino is the same the perimeter may be different. They need to record their results on the activity worksheet. This can also be done with a set amount of yellow square pattern blocks. Students can manipulate them into different shapes to see that no matter how you arrange them the area will stay the same. The teacher should ask students to use the set of pattern blocks to find the smallest perimeter and the biggest perimeter.

Pentominoes Worksheet

Name ________________________

F perimeter_____ area_____

L perimeter_____ area_____

I perimeter_____ area_____

N perimeter_____ area_____

P perimeter_____ area_____

T perimeter_____ area_____

U perimeter_____ area_____

V perimeter_____ area_____

W perimeter_____ area_____

Y perimeter_____ area_____

Z perimeter_____ area_____

Student Lesson: "Puppy Pen"

Description: Students use centimeter graph paper to find different areas using the same perimeter.

Learner Outcomes: Students will look for relationships between perimeter and area through activities that address the following concepts:

Perimeter

- Transitivity
- Conservation of length

Area

- Conservation
- Structuring of an array

Materials:

- Centimeter graph paper

Estimated time needed for lesson: 30-40 minute period

Introduce this lesson to students with the following problem: A family used a fence to make a rectangular-shaped pen for their puppies. A week later, they discovered they had to move the pen to another area of their yard. They used the entire length of the exact same fence, but they made a square-shaped pen.

Ask students the following questions:

Do the puppies have more room to play?

Do they have less room to play?

Do they have the same amount of room to play?

Allow students to work with centimeter grid paper to create the two fences. Be sure to use the same amount of “material” to make each fence, for example forty feet. Students need to determine the area within each fence and record their results. The discussion should reiterate that the fact that even though the perimeter stays the same, the area can change. Students can also show which puppy pen was the smallest and biggest.

Student Lesson: “Fenced In”

Description: Students use centimeter graph paper to find different areas using the same perimeter.

Learner Outcomes: Students will look for relationships between perimeter and area through activities that address the following concepts:

Perimeter

- Transitivity
- Conservation of length

Area

- Conservation
- Structuring of an array

Materials:

- Plain white paper

Estimated time needed for lesson: 50-60 minute period

Students are asked to find different pastures that can be created using the same amount of fencing material. The question asked is, “What is the largest pasture that a rancher can build using the same amount of fencing material?” Students need to show examples of the smallest and biggest pastures that can be created.

APPENDIX C

PRE- AND POST-ASSESSMENTS

Pre-assessment

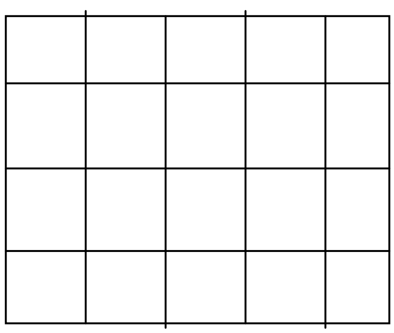
The pre-assessment used in this method checks for whether a student can: (a) identify situations that require using perimeter or area, (b) find the perimeter and area of squares and rectangles that are filled with squared centimeters and those that are not, (c) find the perimeter and/or area of a figure described by words, and (d) draw a shape that possesses a certain perimeter and area.

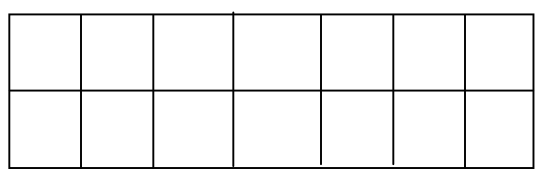
Name ____________________

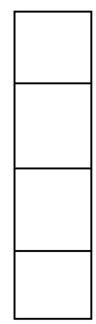
a. Perimeter and Area Identification: Complete the sentences using the words perimeter or area.

1. To know which carpet is the biggest, compare the _______________.
2. To know which field will need the most fencing, compare the _____________.
3. Find the ______________ of a wall to know how much wallpaper will cover it.
4. Find the ______________ of scarf to know how much fringe will go around it.
5. To know how many floor tiles to buy, you must know the _________ of the floor.
6. A road all around the borders of a ranch is called a _______________ road.

b. Perimeter Part One: What is the perimeter of the figure if each side of a square in the figure is one centimeter long? Please label correctly!



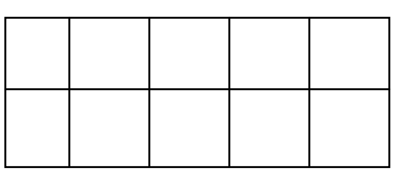


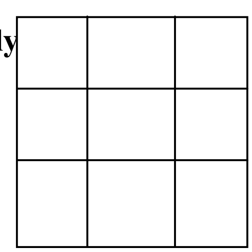


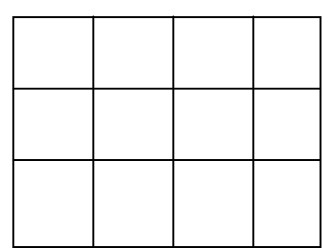
2. Perimeter = ________

1. Perimeter = __________ 3. Perimeter = ______

c. Area Part One: What is the area of the figure if each square is one square centimeter (cm^2)? Please label correctly

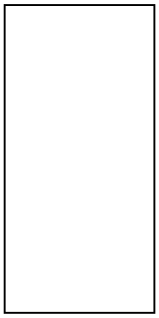




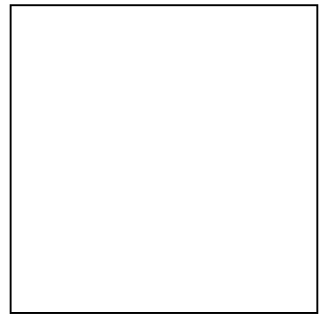


1. Area = __________ 2. Area = __________ 3. Area = ________

d. **Perimeter Part Two: Use a centimeter ruler to find the perimeter of the figure. Please label correctly!**



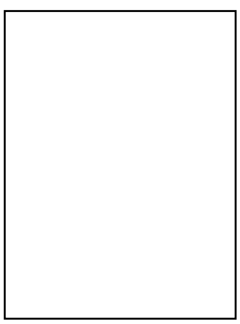
2. Perimeter = __________



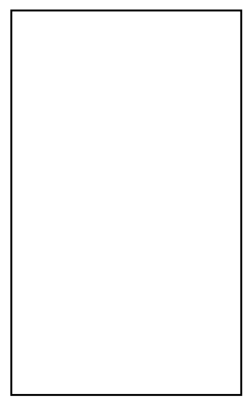
1.Perimeter = __________

3.Perimeter = _________

e. **Area Part Two: Use a centimeter ruler to find the area of the figure. Please label correctly!**



2. Area = ________



1. Area = __________

3. Area = ________

f. **Word Problems: Please answer the following questions.**

1. Tiffany wants new carpeting for her family room. Her family room is a six feet by five feet rectangle. How much carpeting does she need to buy to cover her entire family room? ________________

2. Grace is making a display board for the school talent show. The display board is a six feet by eleven feet rectangle. She needs to add a ribbon border around the entire display board. What is the length of ribbon that she needs? ___________

3. A rectangular dining room is six meters long and three meters wide. What is its area? _______________

4. The perimeter of a rectangular dining room is twenty-six meters. The dining room is five meters wide. How long is it? _______________

g. Perimeter and Area Relationships: On the centimeter grid paper provided, draw a shape that has a perimeter of 20 centimeters and an area of 24 square centimeters.

Post-assessment

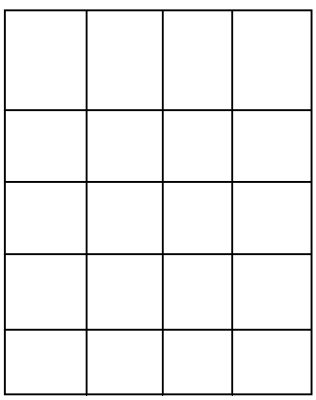
The post-assessment used in this method checks for whether a student can: (a) identify situations that require using perimeter or area, (b) find the perimeter and area of squares and rectangles that are filled with squared centimeters and those that are not, (c) find the perimeter and/or area of a figure described by words, and (d) draw a shape that possesses a certain perimeter and area.

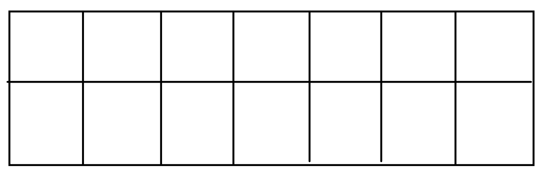
Name ____________________

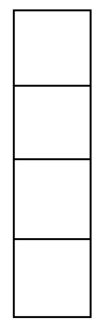
a. **Complete the sentences using the words perimeter or area**.

1. To know which window will need the longest length of lights to go around it, compare the ________________.
2. To know which table has the biggest top, compare the _______________.
3. Find the ______________ of a lake to find how long the walk is around it.
4. Find the ______________ of a wall to know how much wallpaper is needed to cover it.
5. To know how much grass seed to buy, you must know the _______________ of the yard.
6. Find the ______________ to know how much trim is needed to frame a picture.

b. What is the perimeter of the figure if each side of a square in the figure is one centimeter long? Please label correctly!





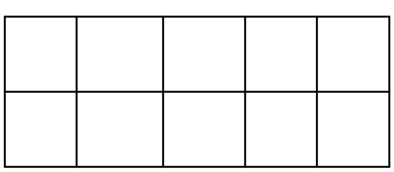


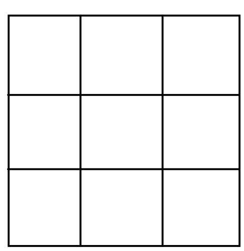
2. Perimeter = __________

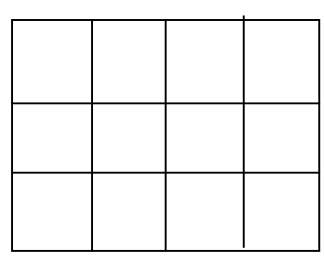
1. Perimeter = __________

3.Perimeter = _________

c. What is the area of the figure if each square is one square centimeter (cm^2)? Please label correctly



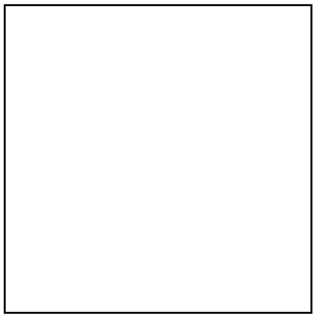




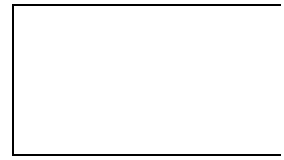
1. Area = __________

2. Area = _________ 3. Area = _________

d. Use a centimeter ruler to find the perimeter of the figure. Please label correctly!



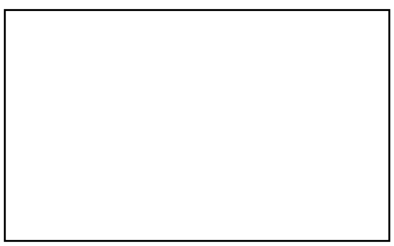
2. Perimeter = __________



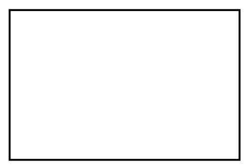
1. Perimeter = _________

3. Perimeter = ____

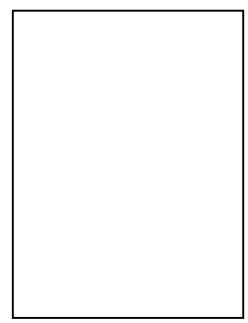
e. Use a centimeter ruler to find the area of the figure. Please label correctly!



1. Area = ________



2. Area = __________



3.Area = _______

f. Please answer the following questions.

1. Tim wants flooring for his living room. His living room is a 20 ft by 10 ft rectangle. How much flooring does he need to buy to cover his entire living room? ________________
2. Greg is making a display board for the school Science Fair. The display board is a four feet by three feet rectangle. He needs to add a striped border around the

entire display board. What is the length of striped border that he needs?

____

3. A rectangular dog pen is five meters long and four meters wide. What is its area?

____

4. The perimeter of a rectangular cafeteria is 120 meters. The cafeteria is 20 meters wide. How long is it? ________________

g. On the centimeter grid paper provided, draw a shape that has a perimeter of 24 centimeters and an area of 32 square centimeters.

APPENDIX D

ITEM BY STUDENT DATA

Subscale Data from Pre-Assessment: (a) Perimeter and Area Identification

	Question 1	Question 2	Question 3	Question 4	Question 5	Question 6
Student 1	0	0	0	0	0	0
Student 2	1	1	1	1	1	1
Student 3	1	1	1	1	1	1
Student 4	0	0	0	0	0	0
Student 5	1	1	1	1	1	1
Student 6	0	0	0	0	0	0
Student 7	1	1	0	0	0	0
Student 8	1	0	0	1	1	1
Student 9	0	1	1	0	1	1
Student 10	0	0	0	0	0	0
Student 11	0	0	0	0	0	0
Student 12	1	1	1	1	1	1
Student 13	0	0	0	0	0	1
Student 14	0	0	1	1	1	0
Student 15	0	0	1	1	1	0
Student 16	1	1	0	0	1	0
Student 17	0	0	0	0	0	0
Total	7	7	7	7	9	7

Subscale Data from Pre-Assessment: (b) Perimeter Part One and (c) Area Part One

	Question 1	Question 2	Question 3	Question 1	Question 2	Question 3
Student 1	1	0	1	1	1	0
Student 2	1	1	1	0	0	0
Student 3	1	1	0	1	1	0
Student 4	0	0	0	0	0	0
Student 5	0	0	0	1	1	1
Student 6	0	0	0	0	0	0
Student 7	0	0	0	0	0	0
Student 8	1	1	1	0	0	0
Student 9	0	0	0	0	1	1
Student 10	0	0	0	1	0	0
Student 11	1	1	1	0	0	0
Student 12	1	1	1	1	1	1
Student 13	1	1	1	0	0	0
Student 14	0	0	0	1	1	1
Student 15	0	0	0	0	0	0
Student 16	1	1	1	0	0	0
Student 17	0	0	0	0	0	0
Total	8	7	7	6	6	4

Subscale Data from Pre-Assessment: (d) Perimeter Part Two and (e) Area Part Two

	Question 1	Question 2	Question 3	Question 1	Question 2	Question 3
Student 1	0	1	1	1	0	0
Student 2	1	1	0	0	0	0
Student 3	1	1	1	1	1	1
Student 4	0	0	0	0	0	0
Student 5	0	0	1	1	1	1
Student 6	0	0	0	0	0	0
Student 7	0	0	1	0	0	0
Student 8	1	1	0	0	0	0
Student 9	0	0	0	1	0	1
Student 10	0	0	0	0	1	0
Student 11	1	1	1	0	0	0
Student 12	1	1	1	1	1	1
Student 13	1	1	1	0	0	0
Student 14	0	0	0	0	0	0
Student 15	0	0	0	0	0	0
Student 16	1	1	1	0	0	0
Student 17	0	0	0	0	0	0
Total	7	8	8	5	4	4

Subscale Data from Pre-Assessment: (f) Word Problems and (g) Perimeter and Area Relationships

	Question 1	Question 2	Question 3	Question 4	Question 1
Student 1	0	1	1	0	0
Student 2	0	1	1	0	0
Student 3	0	0	1	0	0
Student 4	0	0	0	0	0
Student 5	1	0	1	0	0
Student 6	0	0	0	0	0
Student 7	1	0	1	0	0
Student 8	1	0	1	0	0
Student 9	0	0	1	0	0
Student 10	1	0	0	0	0
Student 11	0	0	0	0	0
Student 12	1	1	1	1	1
Student 13	0	0	0	0	0
Student 14	1	0	1	0	0
Student 15	0	0	0	0	0
Student 16	0	0	0	0	0
Student 17	0	0	1	0	0
Total	7	3	10	1	1

Subscale Data from Post-Assessment: (a) Perimeter and Area Identification

	Question 1	Question 2	Question 3	Question 4	Question 5	Question 6
Student 1	1	1	1	1	1	1
Student 2	1	0	1	1	1	1
Student 3	1	1	1	1	1	1
Student 4	1	0	1	1	0	1
Student 5	1	1	1	1	1	1
Student 6	0	1	0	0	0	0
Student 7	1	1	1	1	1	1
Student 8	1	1	1	1	1	1
Student 9	1	1	1	1	1	1
Student 10	1	1	1	1	1	1
Student 11	0	0	0	0	0	0
Student 12	1	1	1	1	1	1
Student 13	1	1	1	1	1	1
Student 14	0	0	1	1	0	0
Student 15	0	1	1	0	1	1
Student 16	1	1	1	1	1	0
Student 17	1	1	0	0	1	0
Total	13	13	14	13	13	12

Subscale Data from Post-Assessment: (b) Perimeter Part One and (c) Area Part One

	Question 1	Question 2	Question 3	Question 1	Question 2	Question 3
Student 1	1	1	1	1	1	1
Student 2	1	1	1	1	1	1
Student 3	0	0	0	1	1	1
Student 4	1	1	1	1	1	1
Student 5	1	1	1	1	1	1
Student 6	0	0	0	0	0	0
Student 7	0	0	1	0	1	1
Student 8	1	1	1	1	1	1
Student 9	0	0	0	1	1	1
Student 10	1	1	1	1	1	1
Student 11	1	1	1	1	1	1
Student 12	1	1	1	1	1	1
Student 13	1	1	1	1	1	1
Student 14	0	0	0	1	1	1
Student 15	0	0	0	0	0	0
Student 16	0	1	1	1	1	1
Student 17	0	1	1	1	0	1
Total	9	11	12	14	14	15

Subscale Data from Post-Assessment: (d) Perimeter Part Two and (e) Area Part Two

	Question 1	Question 2	Question 3	Question 1	Question 2	Question 3
Student 1	1	1	1	1	1	1
Student 2	1	1	1	0	0	0
Student 3	1	0	1	1	1	1
Student 4	1	1	1	1	1	1
Student 5	1	0	1	1	1	1
Student 6	0	0	0	0	0	0
Student 7	1	1	1	0	0	0
Student 8	1	1	1	1	1	0
Student 9	0	0	0	1	0	1
Student 10	1	1	1	1	1	1
Student 11	1	1	1	0	0	0
Student 12	1	1	1	1	1	1
Student 13	1	1	1	1	1	1
Student 14	0	0	0	0	0	0
Student 15	0	0	0	0	0	0
Student 16	0	0	0	1	1	1
Student 17	0	0	0	0	0	1
Total	11	9	11	10	9	10

Subscale Data from Post-Assessment: (f) Word Problems and (g) Perimeter and Area Relationships

	Question 1	Question 2	Question 3	Question 4	Question 1
Student 1	1	0	1	0	0
Student 2	0	1	0	0	0
Student 3	1	1	0	0	0
Student 4	0	0	0	0	0
Student 5	1	1	1	0	0
Student 6	0	0	0	0	0
Student 7	0	0	0	0	0
Student 8	1	0	0	0	0
Student 9	0	0	1	0	0
Student 10	1	1	1	0	0
Student 11	0	0	0	0	0
Student 12	1	1	1	1	1
Student 13	1	1	1	0	0
Student 14	0	0	1	0	0
Student 15	0	0	0	0	0
Student 16	1	0	1	0	0
Student 17	0	0	0	0	0
Total	8	6	8	1	1