The Game F-Saturator

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Honors Program

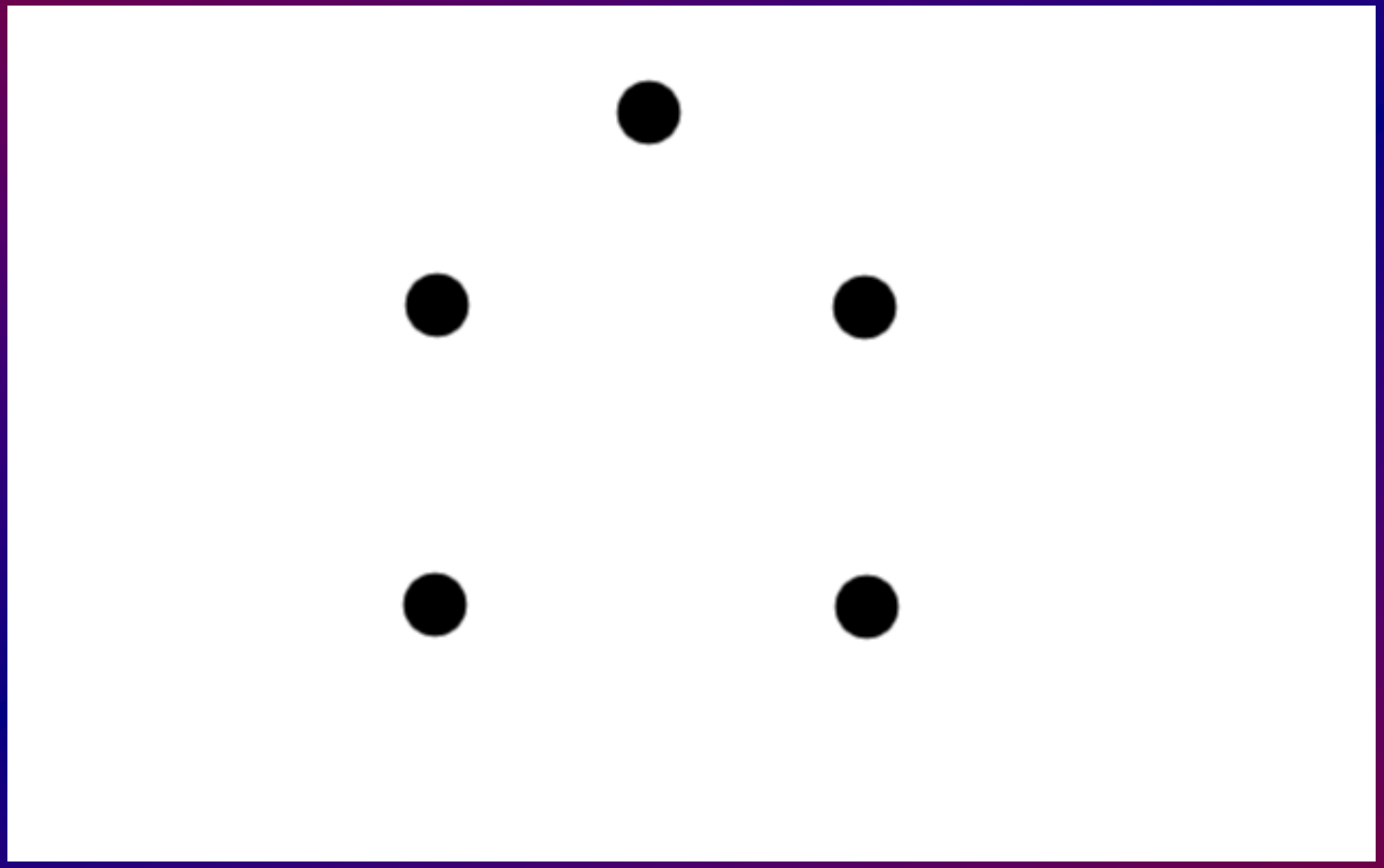
The Rules

- There are two players
- They start with some dots drawn on a piece of paper
- The players take turns connecting the dots. Two dots can only be connected one time.
- A player loses when they are forced to make some shape that has been disallowed. Note that a shape is defined by the connections between the dots, not the shape created by the edges connecting the dots.

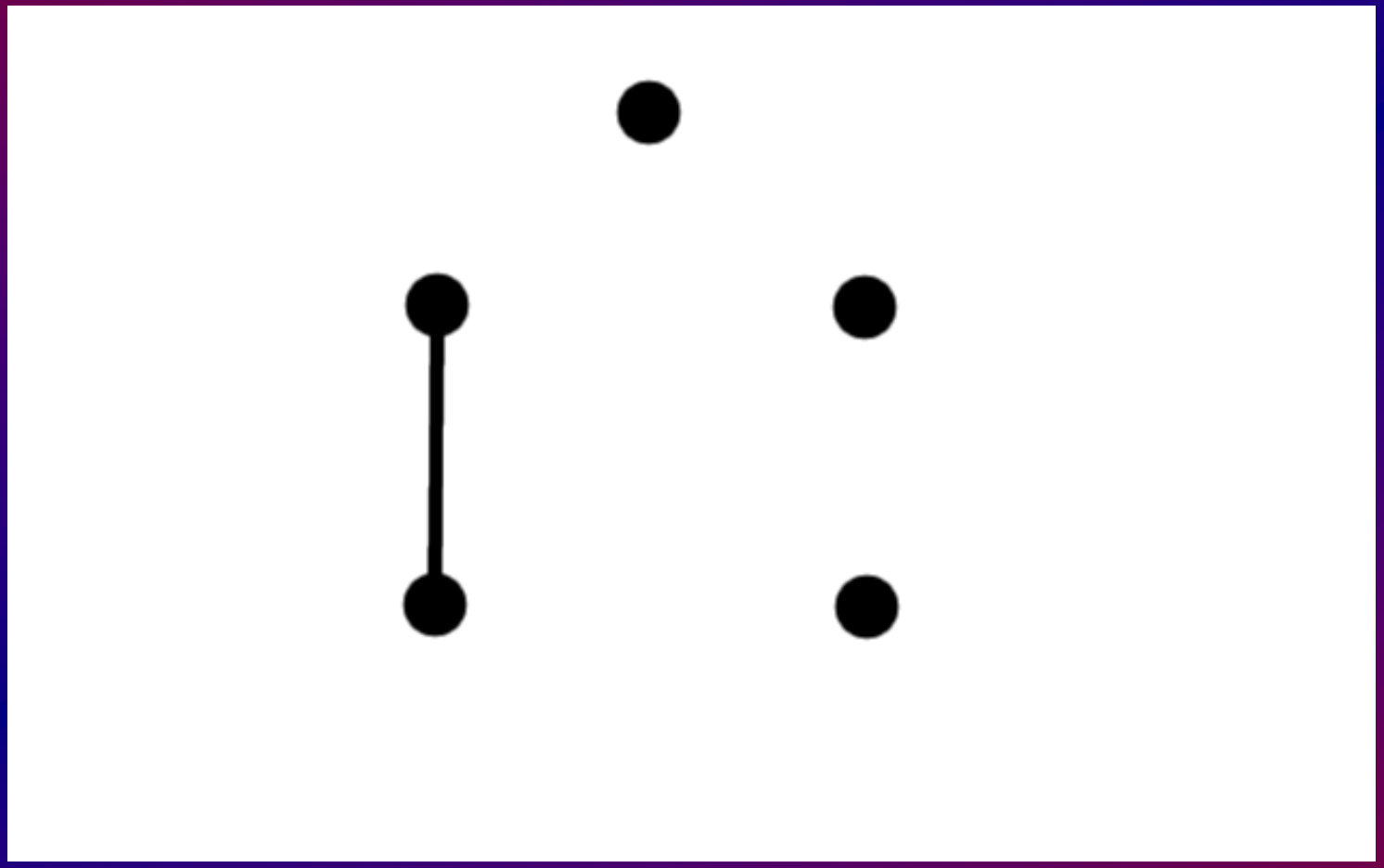
Example Game

- The following is an example game where a triangle is the shape that is not allowed.

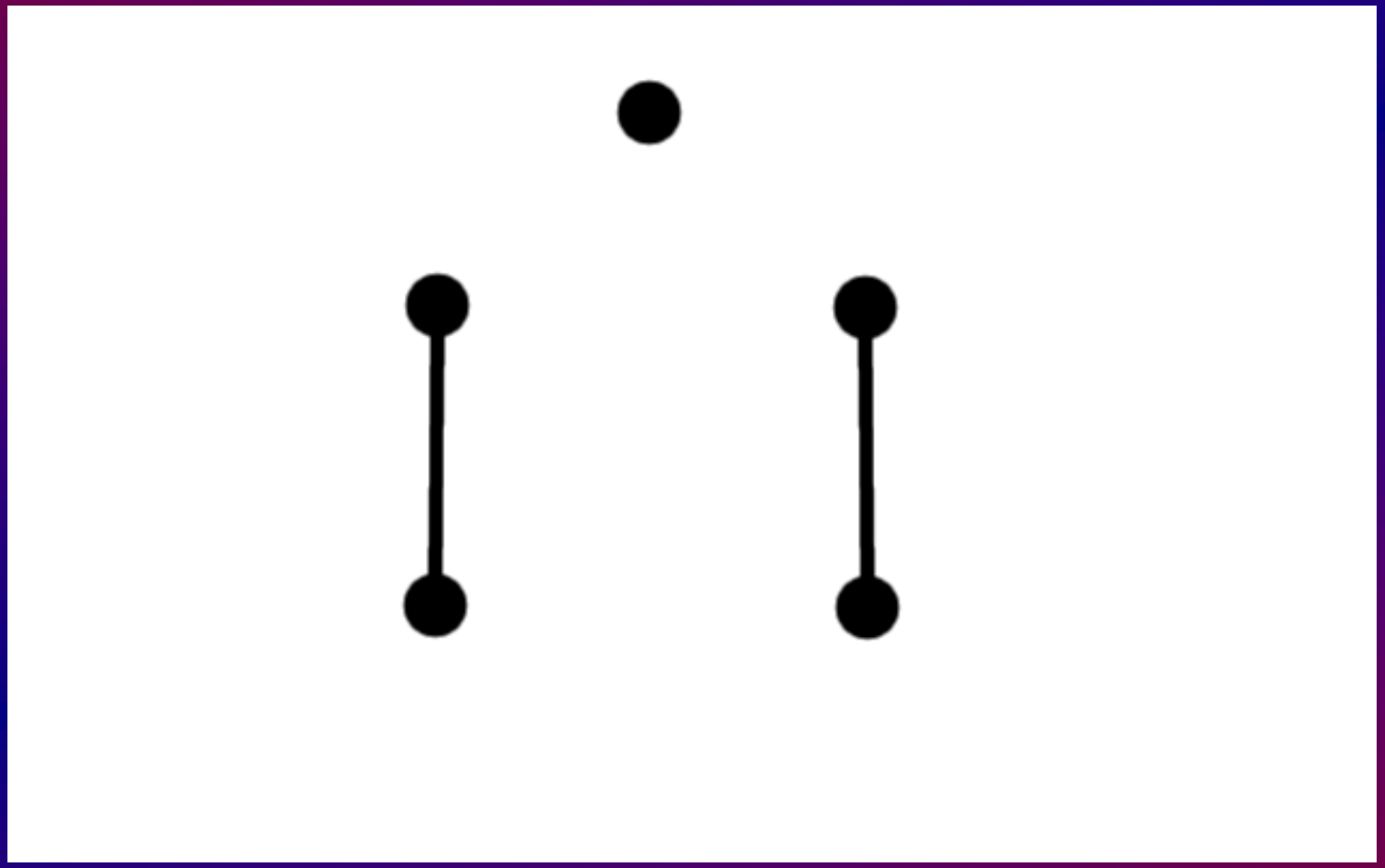
Example Game



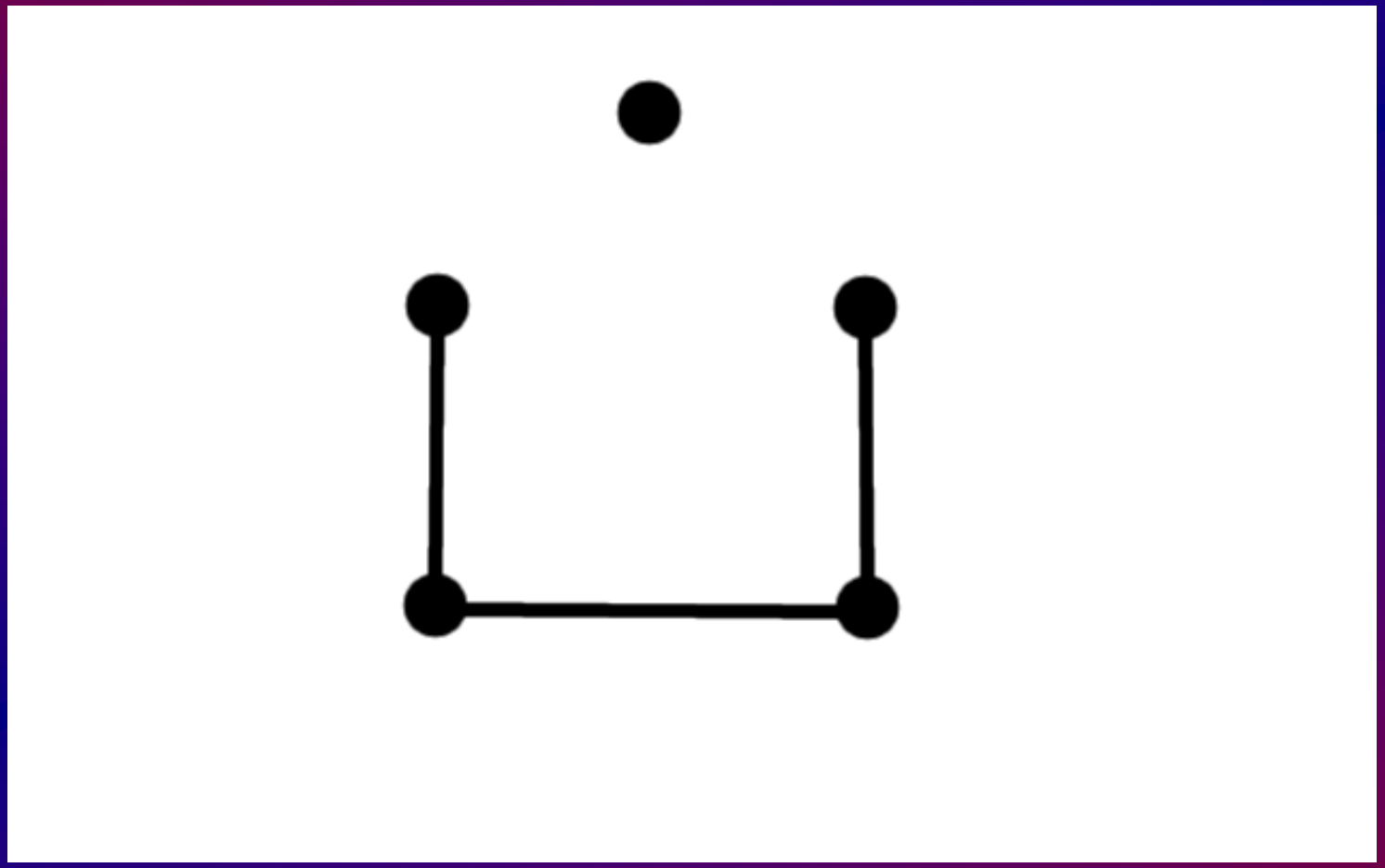
Example Game: Move 1



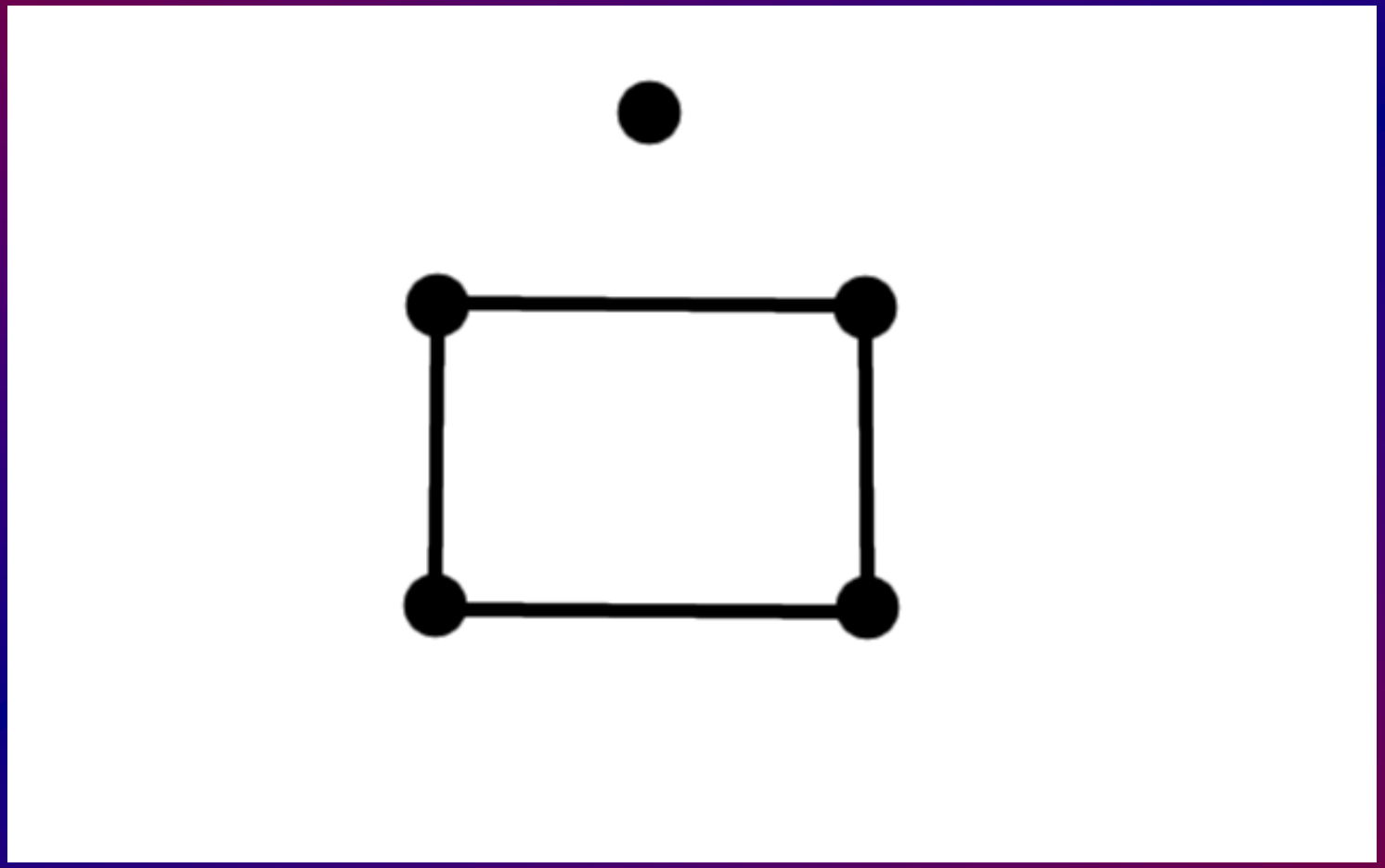
Example Game: Move 2



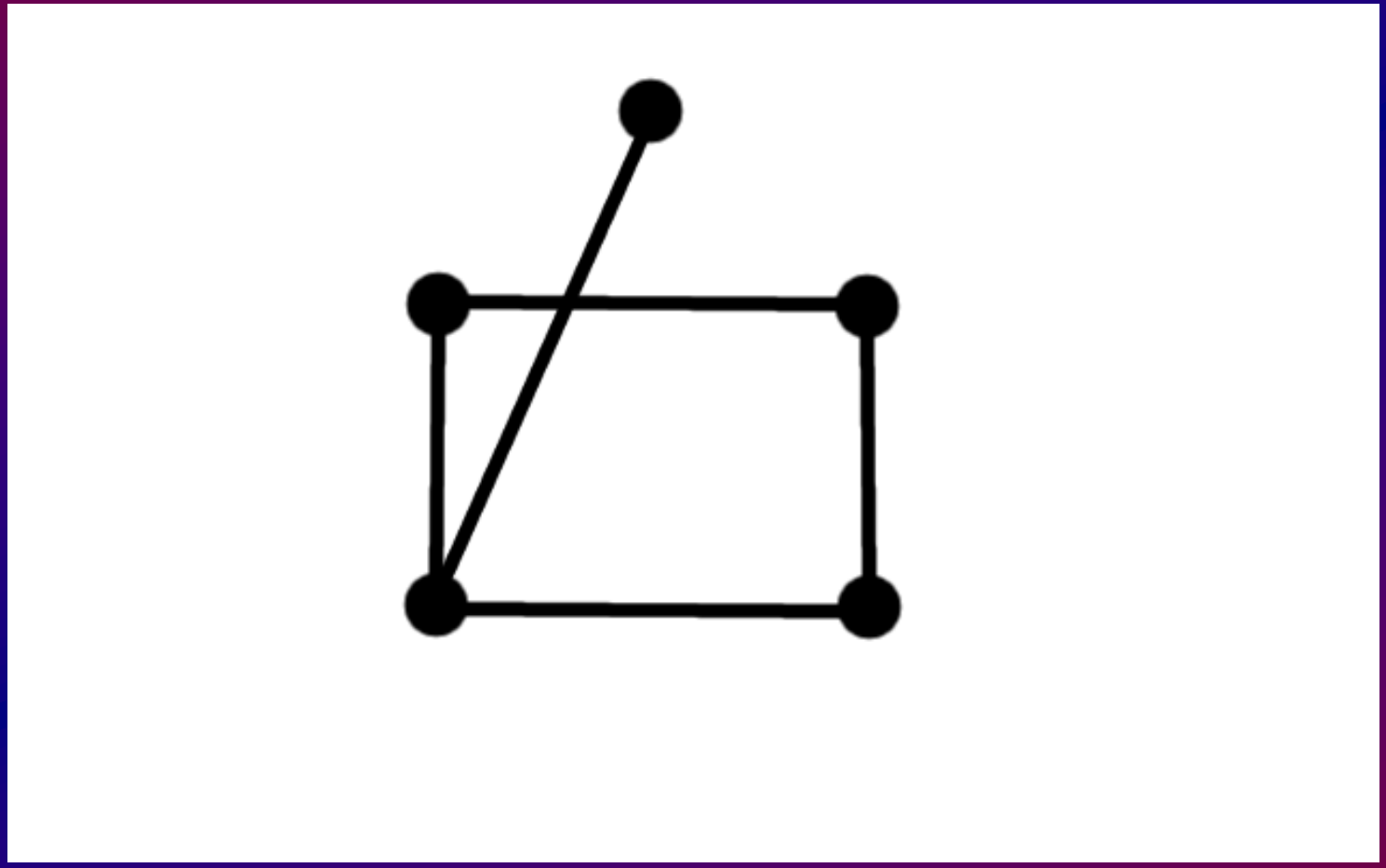
Example Game: Move 3



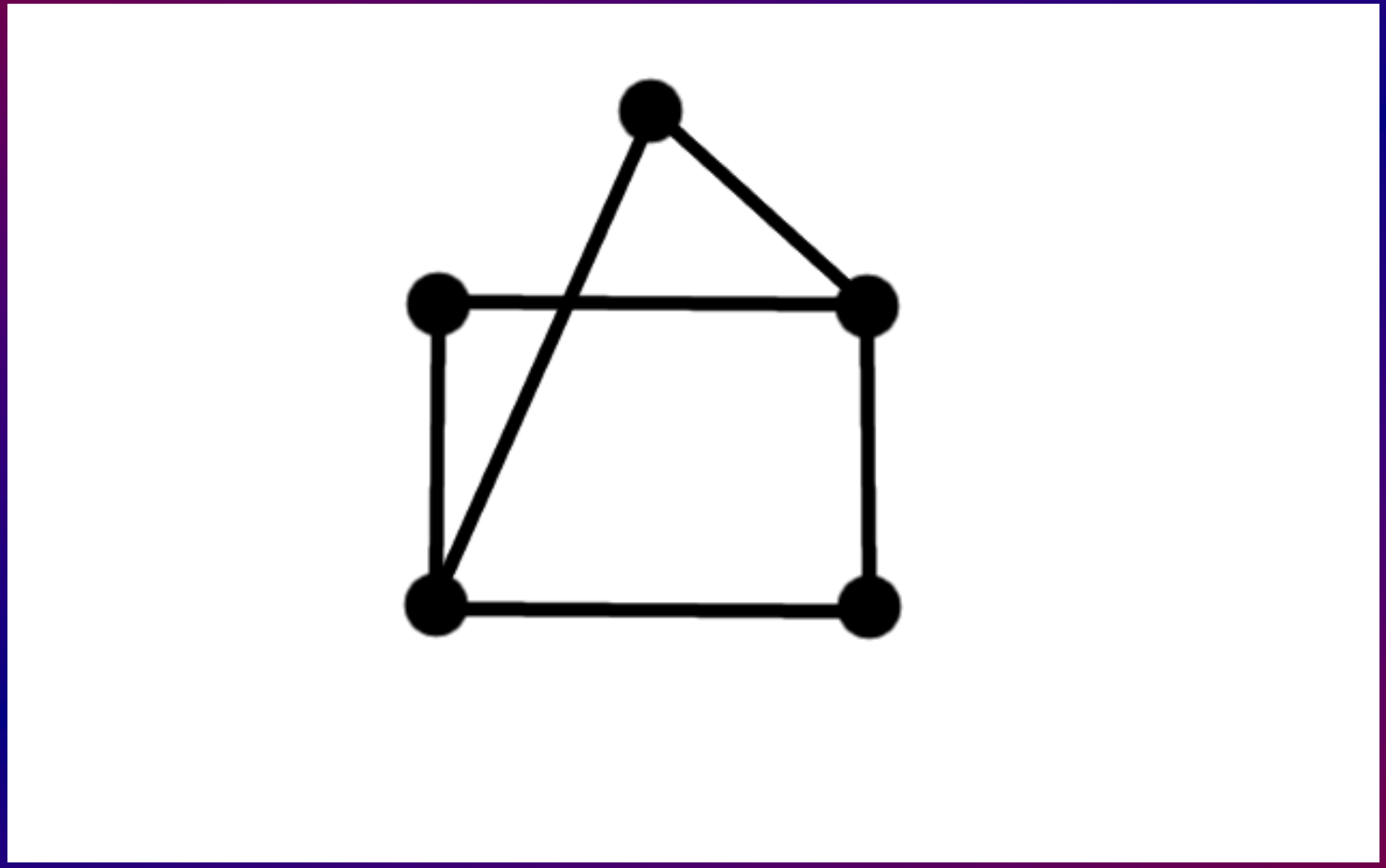
Example Game: Move 4



Example Game: Move 5



Example Game: Move 6



Some Definitions

- Graph: A Graph is a set of vertices (dots) and edges (lines) connecting those vertices.
- Subgraph: A Subgraph is a graph made up of some subset of the vertices of a graph and edges which connect those vertices.

With these definitions, it can be seen that F-Saturator is played by adding edges to a graph where the players are attempting to avoid some subgraph.

The Purpose of this Research

- F-Saturator is a type of game known as a combinatorial game.
- Combinatorial Games have the property that one of the players, if they play correctly, is guaranteed to win.
- The point of this research was to determine which player was guaranteed to win and what strategies could be used.

Why is this Useful?

- Graphs are used to model many phenomenon. Some examples are networks, such as the internet. Graphs can model more abstract concepts as well, such as a game.
- Since graphs are useful for modeling, it can be enlightening to understand graphs which have some special properties.

The Reason for this Game

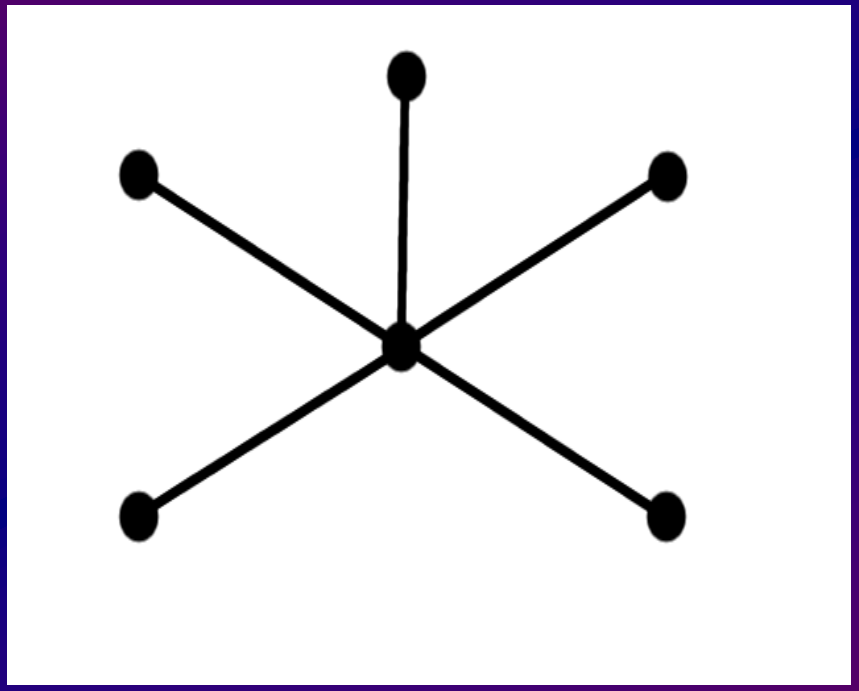
- This game is meant to explore properties of saturated graphs which is why it was first proposed by Ferrara et al.
- An F-Saturated graph G is a graph which does not contain a subgraph F, but adding any edge to G creates a subgraph F.
- The endstates of F-Saturator are all Saturated Graphs. Playing F-Saturator may find some useful properties of F-Saturated graphs..

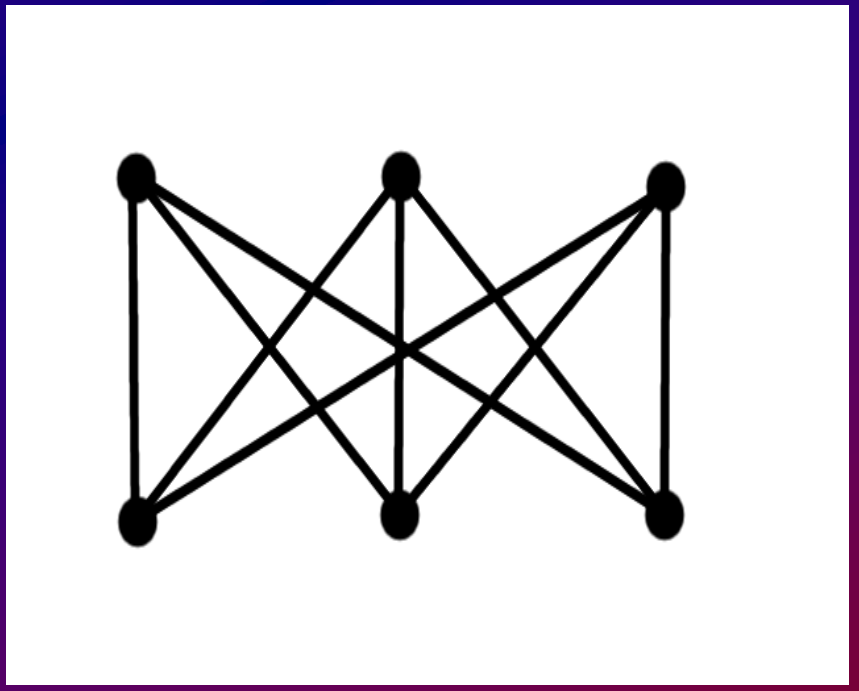
Previous Research

- Much of the previous research on Saturated graphs deals with finding maximal or minimal saturated graphs. These are saturated graphs with the most possible edges and the fewest possible edges.

Examples of Maximal and Minimal Saturated Graphs

- Here is a a minimal triangle saturated graph and a maximal triangle saturated graph.





Hajnal's Triangle Free Game

- The game played in the example is a problem that was originally proposed by Andras Hajnal and existed long before the generalization F-Saturator was proposed.
- There is no known general solution.

Hajnal's Triangle Free Game

- Seress in 1992 found that the first player wins the connected version of this game if and only if the number of vertices is even.
- The original game has been solved through brute force computing on graphs up to 16 vertices by Gordinowicz and Pralat.

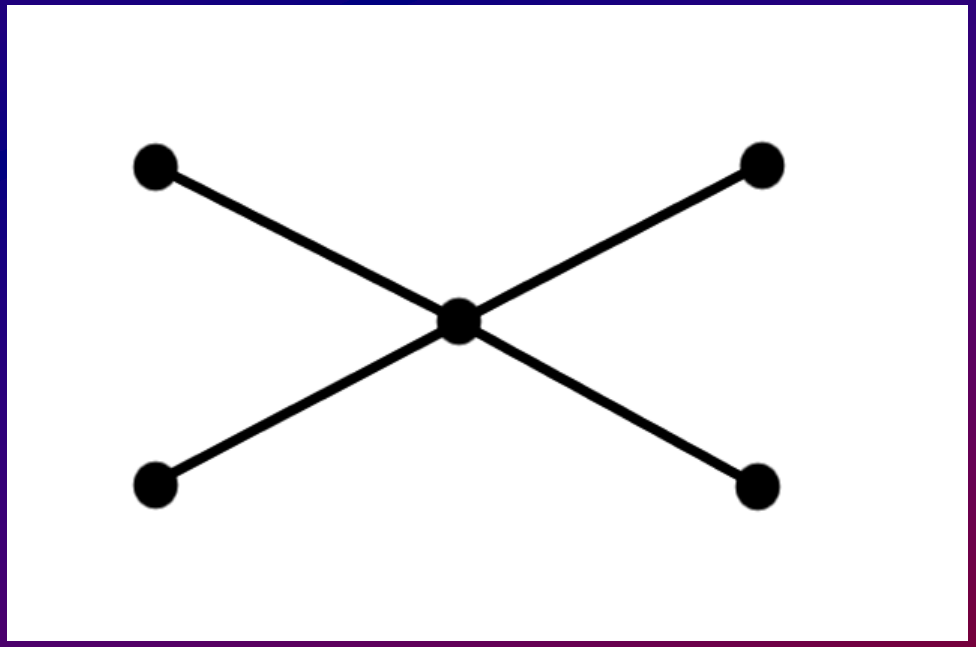
Computed Solutions

Number of Vertices	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Winning Player	2	2	2	1	2	2	2	1	2	1	1	1	1	1

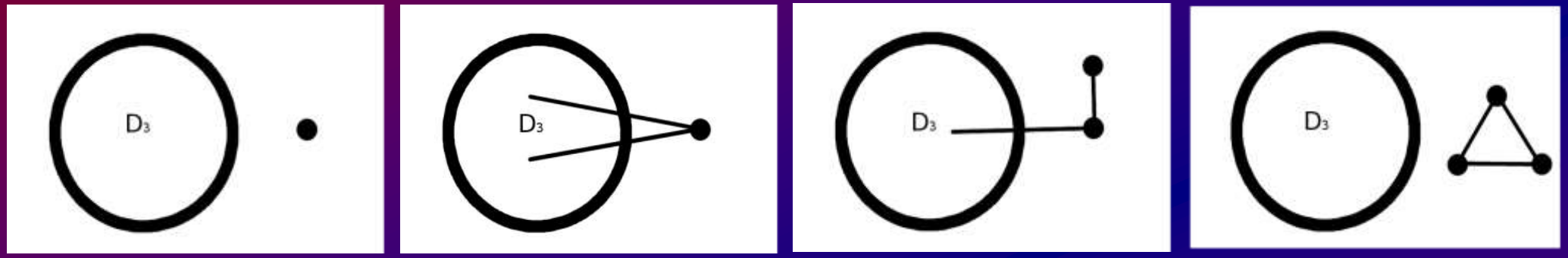
- It took about 1.46 CPU years to compute when there were 16 vertices
- It is estimated it will take more than 46 CPU years when there are 17 vertices

The Research

- Studied S_4-Saturator. This is F-Saturator when the graph being avoided is the one shown in the picture. That graph is called the S_4 graph.



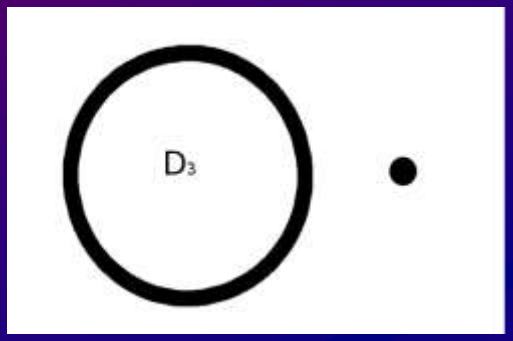
The End States of S_4 Saturator

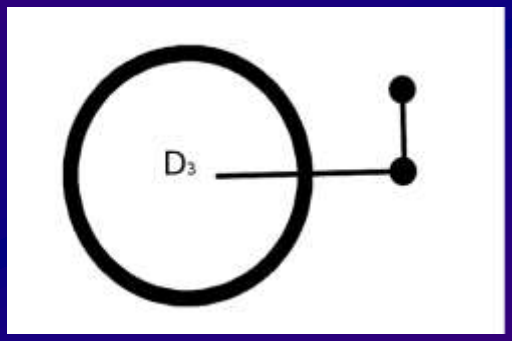


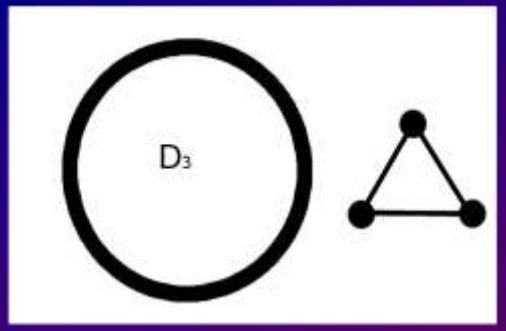
- These are the end states of S_4-Saturator
- The big circle represents a bunch of vertices which are connected to exactly 3 edges.

The States

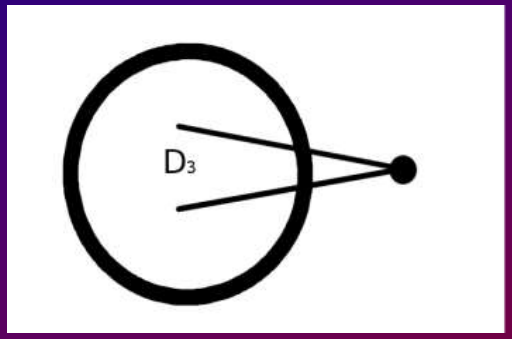
- The same player wins if the game ends in these 3 states.





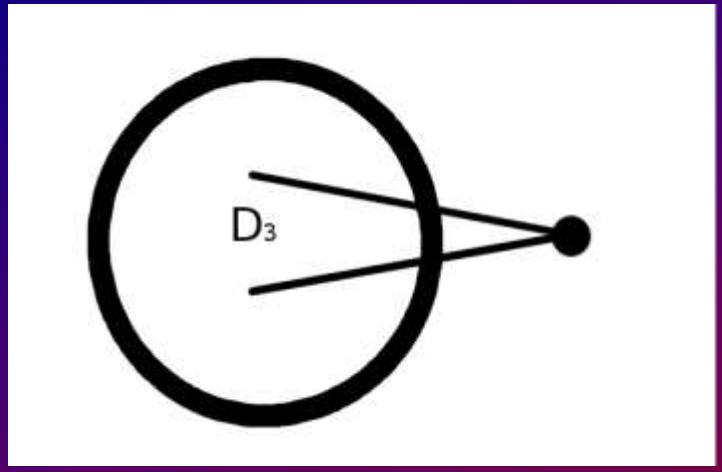


- The other player wins if the game ends in this state.



The Winner

- Even though one of the player has three states they can with with, it turns out that when the game is played perfectly and the number of vertices is odd and greater than or equal to 9, the following state will be the end state.



The Strategy

- The strategy by the winning player is to get every vertex to have an edge connected to it.
- Once all the vertices have been connected, play does not matter until the last 4 moves.
- In the last 4 moves it is always possible to make moves guaranteeing the outcome. The move depends on the case though.

The End

- Thanks to Jason Williford my advisor.

Sources

- M. Ferrara, M. Jacobson, A. Harris. The game of F-saturator. Discrete Appl. Math. 158 (2010), no. 3, 189-197.
- P. Gordinowicz, P. Pralat. The first player wins the one-colour triangle avoidance game on 16 vertices. Discussiones Mathematicae Graph Theory 32 (2012), no. 1, 181-185.