Understanding Before Executing: A Literature Review of Conceptual Fraction Instruction and Its Implications for Upper Elementary Students and Teachers

A LITERATURE REVIEW

By Toni Ullrich

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Masters Committee:

Alison K. Mercier, Assistant Professor, Chair

Michelle T. Chamberlin, Associate Professor

JP Han, Assistant Professor

Abstract

Fractions remain one of the most persistently challenging topics in upper elementary mathematics despite decades of curricular reform. This extended literature review examines the long-term benefits of teaching fractions conceptually before procedurally in upper elementary classrooms, with fraction division serving as the primary focal concept through which this argument is examined, synthesizing research from the 1990s through contemporary systematic reviews. Three interconnected theoretical frameworks guide this analysis: the Conceptual vs. Procedural Knowledge Framework, the Mathematics Achievement Prediction Framework, and the Proficiency Development Framework. Together, these frameworks demonstrate that conceptual-first instruction provides the cognitive foundation for meaningful fraction learning and fraction division understanding specifically, predicts sustained mathematical achievement, and supports knowledge transfer to ratios, proportional reasoning, and algebra. The review also addresses significant counterpoints, including the persistence of whole number bias, which is particularly consequential for fraction division where whole number intuitions about division are most directly contradicted, individual variation in development pathways, and limitations of traditional part-whole instructional approaches. These counterpoints refine rather than undermine the central argument, pointing toward a more nuanced position: procedural instruction alone is demonstrably insufficient, conceptual understanding must come first to initiate meaningful learning, and teachers must remain responsive to individual students as conceptual and procedural knowledge develop in tandem. A consistent finding across the literature is that effective conceptual instruction depends fundamentally on teachers' own conceptual understanding, as many elementary teachers lack the mathematical depth necessary to teach fractions meaningfully. The evidence affirms that improving fraction instruction requires not merely resequencing content, but transforming mathematical experiences through investment in teacher preparation and instructional practices that prioritize conceptual depth over procedural coverage.

Keywords: fractions, conceptual understanding, procedural knowledge, upper elementary mathematics, fraction instruction, mathematical achievement, teacher preparation

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Dedication

This work is dedicated to my mother-in-law, Sue, whose wisdom and spirit continue to guide me in ways I am still discovering. Sue had a rare and beautiful gift for putting life into perspective; she reminded me to be fully present in every moment, to always enjoy the family in my home over the chores in the home, and to always dance in the rain rather than wait for the storm to pass. She also taught me to never lose sight of how deeply blessed I am, especially in the privilege of making a difference in the lives of children. Her belief in the importance of that work, and in me, left a mark I carry into my classroom every single day. She is greatly missed, but her legacy lives on in every moment I choose joy, every time I let go of what does not matter, and every child whose life I am fortunate enough to touch.

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Chapter 1

Introduction

During my time in elementary, middle, and high school, I learned the content that was being taught without questioning whether I was developing a deep enough understanding to make my education valuable. It was not until I began teaching in a fifth-grade classroom, leading the instruction and education of my students, that I realized I lacked a strong conceptual grasp of specific mathematical concepts. I could perform mathematics at a procedural level, utilizing all the memorized formulas, tricks, tips, and acronyms, but my understanding did not extend beyond these surface-level strategies. It quickly became frustrating for me that I could not answer the *why* questions my students were asking, nor could I visually demonstrate what was happening as we learned concepts. Digging into the math curriculum that the district required us to use, I learned some of the conceptual pieces that were missing in my own education. As I began breaking instruction into smaller steps, I was astonished to discover gaps in my own conceptual understanding of mathematics. My experience reflects a broader pattern identified by Copur-Gencturk (2021), who found that many elementary teachers nationwide lack the deep conceptual understanding of fraction operations necessary to teach these concepts meaningfully. My students were spending their days with me learning tricks like "leave it, change it, flip it," and "add a magic zero, take a disappearing zero" instead of learning how to conceptually understand complex division of fractions and why the inverse or zeros are needed to demonstrate value within the place value system. Deciding to earn a master's degree in middle school mathematics was a decision I made wholeheartedly to become a better math teacher for my students and their futures. As door after door opened over the course of two years in the program, I was not only learning math myself, but also learning how to find resources that would benefit me in teaching and learning conceptual ideas with my students. The most challenging task so far has been deciding which of these concepts to research at a deeper level. Fifth-grade content encompasses concepts from place value, decimals, and fractions to volume, area, and graphing, so options are extensive when considering how these concepts can be taught more effectively using a conceptual framework before introducing the procedures students will use long-term. Since our state standardized test heavily relies on knowledge of fractions, accounting for 33% of the overall assessment, I decided to delve into conceptual learning of fractions. After all, I had recently learned a great deal about fractions and had completely changed the way I approach them with students. In particular, fraction division, the operation I had taught for years as "leave it, change it, flip it" became the clearest example of the difference between procedural and conceptual instruction. Once I understood what fraction division actually means, that we are asking how many groups of one fractional quantity fit into another, I could see how little my students had understood and how much more was possible when conceptual understanding came first. For this reason, fraction division serves as the specific focal concept of this review, representing the broader case for why conceptual instruction must precede procedural instruction in upper elementary mathematics.

Statement of the Problem

Despite decades of research and curricular reform, fractions remain one of the most conceptually challenging topics for upper-elementary students (Geary et al., 2008). More than 35 years of research confirms that fraction learning continues to pose significant challenges, with persistent difficulties in understanding part-whole relationships, equivalence, and fraction operations (Syed Ismail et al., 2024). Gabriel and colleagues (2013) identified specific conceptual barriers including understanding part-whole relationships, equivalence, and the density of rational numbers, which help explain why fractions pose such persistent difficulties for learners. Many students learn to manipulate fractions procedurally by memorizing algorithms for addition, subtraction, multiplication, and division without developing a meaningful understanding of what fractions represent (Barkin, 2019; Kazemi & Stipek, 2001). Early research on fraction instruction revealed that students often construct limited mental models of fractions, restricting their understanding to specific interpretations such as part-whole while failing to grasp other critical meanings like quotient, ratio, operator, and measure (Pitkethly & Hunting, 1996). Fraction division is among the most consequential of these operations precisely because it draws on all of these interpretations simultaneously, students must understand fractions as quantities, as quotients, and as measures in order to make sense of what dividing by a fraction means. Yet, fraction division is most commonly taught through the invert-and-multiply algorithm with no conceptual grounding whatsoever (Reeder & Utley, 2017). This imbalance between procedural fluency and conceptual understanding has long-term consequences. Studies show that early fraction competence predicts later success in algebra and overall mathematics achievement (Bailey et al., 2012). However, the traditional instructional focus on procedures often results in short-term performance gains at the expense of long-term mathematical reasoning and the transferability of knowledge (DeWolf et al., 2015).

The problem addressed in this extended literature review is the persistent gap in students' deep understanding of fraction division, which occurs when instruction prioritizes procedural skills over conceptual comprehension. Although research emphasizes the importance of conceptual learning, classroom practices in upper elementary grades often continue to rely on rules and procedures for instruction. This results in students failing to connect fractions to other mathematical ideas, such as ratios, proportional reasoning, and algebraic thinking (Geary et al., 2008; Tsai & Li, 2017). Furthermore, Reeder and Utley (2017) found that prospective elementary teachers themselves struggle to articulate what a fraction is beyond definitions, suggesting that the cycle of procedural-focused instruction continues across generations of teachers and students. This cycle is most visible in fraction division instruction specifically, where the persistence of mnemonics like “keep, change, flip” across generations of classrooms reflects a systemic prioritization of procedural memorization over the conceptual understanding that research consistently shows is necessary for long-term mathematical success.

This extended literature review aims to investigate whether a conceptual approach to teaching fraction division yields measurable long-term benefits for students' later mathematical understanding and achievement compared to a procedural approach, and what that finding means for how fraction division is taught in upper elementary classrooms.

Purpose

The purpose of this extended literature review is to examine the long-term benefits of teaching fractions conceptually before procedurally in upper elementary classrooms, with particular focus on fraction division as the specific operation that most clearly illustrates the consequences of prioritizing procedural instruction over conceptual understanding. Fraction division serves as the primary lens of this review because it is simultaneously the fraction operation most dependent on deep conceptual understanding and the one most consistently taught through memorized procedures without conceptual grounding, making it the most consequential case for examining what is gained when conceptual instruction precedes procedural instruction and what is lost when it does not. Specifically, this review aims to synthesize existing research on how conceptual understanding of fraction division supports mathematical reasoning, knowledge transfer, and future success in more advanced mathematical domains including ratios, proportional reasoning, and algebra. By integrating findings from cognitive, instructional, and experimental studies, this extended literature review aims to highlight pedagogical practices that strengthen students' conceptual understanding of fraction division specifically and promote sustained mathematical growth in upper elementary classrooms and beyond.

Research Questions

This extended literature review was guided by the following research questions:

1. What are the effects of conceptually teaching fractions, particularly fraction division, on students' long-term mathematical understanding compared to procedural instruction?

2. How does conceptual understanding of fractions, including fraction division, aid students' ability to transfer knowledge to later concepts such as ratios, proportional reasoning, and algebra?

3. Which instructional practices promote the most beneficial conceptual understanding of fractions and fraction division specifically in upper elementary classrooms?

Methodology

This extended literature review will use an extended literature review methodology to synthesize foundational research on conceptual and procedural fraction teaching. The review will analyze empirical studies, theoretical frameworks, and educational reports that examine the development, implementation, and outcomes of teaching approaches guided by conceptual understanding in upper elementary classrooms. To ensure comprehensive coverage, the review incorporates research spanning from foundational studies in the 1990s (Pitkethly & Hunting, 1996; Rittle-Johnson et al., 2001) through contemporary systematic reviews and bibliometric analyses (Syed Ismail et al., 2024), allowing examination of both the historical development and current state of fraction instruction research. The methodology draws upon cognitive psychology, mathematics education, and instructional design literature to construct an integrated understanding of how students develop fractional knowledge and how teachers can best support development.

The Unified Conceptual Model presented in this review was developed organically through the synthesis process rather than adopted or adapted from a single existing source. As the literature was reviewed, a color-coded organizational system was used to track how individual studies and findings connected to each of the three research questions guiding this review. Each research question was assigned a color, and as sources were read and annotated, findings were categorized under the research question they most directly addressed. Over time, patterns emerged across these categories that revealed consistent thematic clusters: how instructional sequencing affects conceptual and procedural knowledge development, how fraction competence predicts later mathematical achievement, and how specific instructional practices promote fraction proficiency. These clusters formed the three dimensions of the Unified Conceptual Model. As synthesis continued, a second layer of organization emerged naturally from the intersections among these dimensions, revealing three areas of practical application: instructional sequencing, student outcomes, and teacher practices. These three areas sit at the convergence point of all three dimensions in Figure 1. The model was therefore not generated by or copied from any external source but constructed inductively from the research itself, representing this review's own integrative contribution to the literature on conceptual fraction instruction.

Chapter 2

Literature Review

Introduction

This chapter explores the literature related to the conceptual teaching of fractions in upper elementary mathematics, with particular focus on fraction division as the specific operation that most consequentially illustrates what is gained when conceptual understanding precedes procedural instruction and what is lost when it does not, emphasizing how conceptual instruction supports long-term mathematical understanding, transfer of knowledge, and the development of fraction proficiency. Three research questions guide this review: (1) What are the effects of conceptually teaching fractions on students' long-term mathematical understanding compared to procedural instruction? (2) How does conceptual understanding of fractions aid students' ability to transfer knowledge to later concepts such as ratios, proportional reasoning, and algebra? (3) Which instructional practices promote the most beneficial conceptual understanding of fractions in upper elementary classrooms?

The organization of this chapter follows the three interrelated dimensions of the Unified Conceptual Model established in the theoretical foundation: Dimension 1 (Conceptual vs. Procedural Knowledge), Dimension 2 (Mathematical Achievement Prediction), and Dimension 3 (Proficiency Development Through Instructional Practice), each of which directly addresses one of the three research questions guiding this review. Throughout each dimension, fraction division serves as the specific conceptual lens through which the broader argument for conceptual-first instruction is examined and applied. Together, these dimensions provide an integrated understanding of how conceptual fraction instruction shapes students' mathematical growth.

Defining Conceptual and Procedural Understanding in Fraction Learning

Before examining the Unified Conceptual Model and its three interrelated dimensions, it is important to establish a clear and shared understanding of what conceptual and procedural knowledge mean in the context of fraction learning, and how these two forms of knowledge relate to one another in practice. These definitions are not merely academic distinctions; they shape how instruction is designed, sequenced, and evaluated in upper elementary classrooms.

Conceptual knowledge refers to a deep understanding of mathematical ideas and the relationships between them (Schneider et al., 2011). In the context of fraction division, conceptual understanding means that a student can do more than produce a correct answer. It means that the student can represent the problem visually, explain what is happening mathematically, and justify why a particular approach makes sense. For example, a student with conceptual understanding of dividing $\frac{3}{4} \div \frac{1}{2}$ does not simply invert and multiply. Instead, that student can draw an area model or use a number line to show how many groups of $\frac{1}{2}$ fit into $\frac{3}{4}$, articulate what the quotient represents in a real-world context, and explain why the algorithm produces the result it does. Rešić and colleagues (2016) argued that effective fraction instruction requires a deliberate progression from concrete representations through pictorial models to abstract symbolic manipulation. This representational progression is central to what conceptual understanding looks like in practice: students move from physically partitioning quantities to sketching diagrams to writing symbolic expressions, with each stage grounded in meaning rather than memorization.

Procedural knowledge, by contrast, involves knowing how to carry out specific computational steps (Schneider et al., 2011). In fraction division, this is the familiar algorithm:

invert the divisor and multiply. Procedural knowledge is not without value, as fluency with algorithms supports efficiency and accuracy. However, when procedural knowledge develops without conceptual grounding, students often struggle to recognize when an algorithm applies, interpret what their answer means, or recover when they make an error (Lenz et al., 2024). Many students learn the “leave it, change it, flip it” mnemonic for fraction division without ever developing a mental image of what division by a fraction represents or why the inverse operation is mathematically valid (Barkin, 2019).

The critical insight from decades of research is that conceptual understanding is not simply a precursor to procedural fluency; it is the foundation that allows procedural knowledge to be meaningful, flexible, and transferable. Byrnes and Wasik (1991) established that students with stronger conceptual foundations acquire procedures more readily and with greater accuracy than those who begin with procedures alone. Rittle-Johnson and colleagues (2001) further demonstrated that conceptual and procedural knowledge develop through a reciprocal, iterative relationship in which gains in one domain support gains in the other. Despite this bidirectional relationship, their research affirms that initial conceptual grounding provides the most robust scaffold for long-term mathematical development. When students can visually represent and explain fraction division before they are asked to execute the algorithm, they develop a schema that allows them to reason through novel problems rather than rely solely on memorized steps (Barkin, 2019). Geller, Son, and Stigler (2017) similarly found that students who were asked to provide conceptual explanations for their fraction solutions demonstrated deeper and more durable understanding than those who simply calculated answers. Conceptual understanding functions as both the entry point and the sustaining foundation for meaningful fraction learning, including the complex and often misunderstood operation of fraction division.

Unified Conceptual Model

This extended literature review is organized around a Unified Conceptual Model comprising three interrelated dimensions (see Figure 1): The first dimension, Conceptual vs Procedural Knowledge, addresses Research Question 1 by examining how instructional sequencing shapes students' long-term mathematical understanding, with fraction division serving as the primary illustrative case. The second dimension, Mathematics Achievement Prediction, addresses Research Question 2 by establishing fraction competence as a predictor of knowledge transfer to ratios, proportional reasoning, and algebra. The third dimension, Proficiency Development Through Instructional Practice, addresses Research Question 3 by articulating how specific classroom practices promote the most beneficial conceptual understanding. Together, these three dimensions are synthesized from existing cognitive, instructional, and experimental research to explain how and why teaching fractions conceptually before procedurally and fraction division specifically in upper elementary classrooms produces sustained mathematical growth and long-term achievement benefits.

Dimension 1: Conceptual vs Procedural Knowledge forms the foundational layer of this Unified Conceptual Model and directly addresses Research Question 1: What are the effects of conceptually teaching fractions on students' long-term mathematical understanding compared to procedural instruction? This dimension advances a clear instructional position: conceptual understanding must be established before procedural instruction if students are to develop meaningful, lasting mathematical knowledge. This position is most consequentially demonstrated in the context of fraction division, where the invert-and-multiply algorithm is among the most commonly memorized but least conceptually understood procedures in upper elementary mathematics (Reeder & Utley, 2017; Copur-Gencturk, 2021).

Conceptual understanding involves connecting meaning to fraction representation, real-world application, and mathematical relationships, whereas procedural knowledge emphasizes computational and algorithmic fluency (Lenz et al., 2024). These two forms of knowledge are not equivalent in their instructional priority. Early research by Byrnes and Wasik (1991) established that conceptual knowledge plays a facilitative role in mathematical procedural learning, demonstrating that students with stronger conceptual foundations acquire procedures more readily and with greater accuracy than those who begin with procedures alone. This foundational finding is critical: conceptual understanding does not simply accompany procedural skill, it enables it.

The consequences of reversing this sequence are well documented. When procedural instruction precedes conceptual grounding, students develop fragmented understanding that leads to errors and limited transfer of knowledge (Lenz et al., 2024). In fraction division specifically, students who learn to invert and multiply before understanding what division by a fraction means cannot interpret their answers, recognize their errors, or connect the operation to the ratios and proportional reasoning that follow in later grades. Conversely, when conceptual understanding is established first, students develop cognitive scaffolding that supports both procedural efficiency and mathematical reasoning (Barkin, 2019). Siegler and Lortie-Forgues (2015) reinforced this finding, demonstrating that students' conceptual understanding of fraction algorithms work was more strongly associated with problem-solving success than their ability to execute procedures alone.

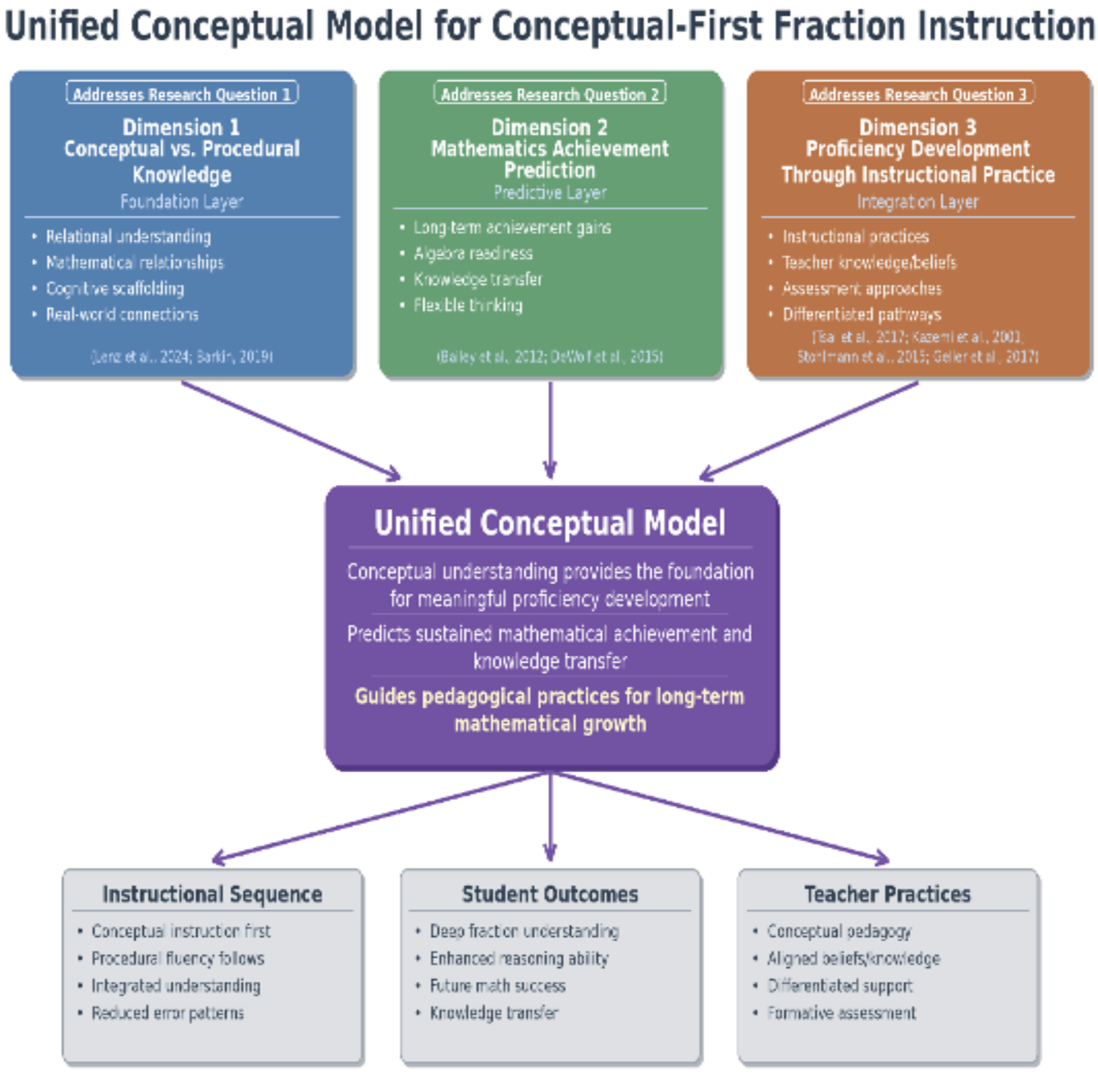
While research does confirm that conceptual and procedural knowledge have a bidirectional relationship, with each form of knowledge capable of reinforcing the other over time (Rittle-Johnson et al., 2001), this iterative dynamic does not mean the two are interchangeable as starting points. Rittle-Johnson, Schneider, and Star (2015) clarified that even within this reciprocal relationship, initial conceptual grounding consistently produces the most robust long-term mathematical development. In other words, the bidirectional relationship is real, but it functions best when conceptual understanding initiates the cycle. Establishing procedures first without conceptual grounding produces a weaker foundation that limits how far the iterative relationship can develop. This foundational distinction is essential for understanding how instructional sequencing shapes student learning outcomes and why conceptual-first instruction is not simply one valid option among many, but the sequence most strongly supported by the evidence.

Dimension 2: Mathematics Achievement Prediction directly addresses Research Question 2: How does conceptual understanding of fractions aid students' ability to transfer knowledge to later concepts such as ratios, proportional reasoning, and algebra? Before examining fractions specifically, it is important to understand what the broader research establishes about conceptual understanding and long-term mathematics achievement. Research consistently demonstrates that students who develop deep conceptual understanding of mathematical topics, rather than relying solely on procedural execution, show stronger long-term achievement outcomes, greater problem-solving flexibility, and more successful transfer of knowledge to new and increasingly complex domains (Geary et al., 2008). Conceptual understanding equips students with the relational reasoning and cognitive structures necessary to connect mathematical ideas across topics, rather than treating each concept as an isolated set of rules to memorize.

Within this broader pattern, fraction competence occupies a uniquely important and well-documented position. This review applies the Mathematics Achievement Prediction framework specifically to fraction learning because the research identifies fraction understanding as one of the strongest individual predictors of later mathematical success (Bailey et al., 2012). Early conceptual understanding of fractions including fraction division generates long-term achievement gains extending into algebra, rational numbers, and advanced mathematical domains, precisely because fractions require students to reason about relationships between quantities in ways that directly underpin proportional and algebraic thinking (Bailey et al., 2012). Beyond computational accuracy, conceptual fraction knowledge supports relational reasoning, knowledge transfer across number systems, and flexible mathematical thinking (DeWolf et al., 2015). Siegler and Lortie-Forgues (2015) further demonstrated that students' conceptual understanding of why fraction algorithms work was more strongly associated with problem-solving success than their ability to execute procedures alone. This review applies these findings to argue that conceptual-first fraction instruction in upper elementary classrooms is not simply a pedagogical preference, but an evidence-based investment in students' long-term mathematical trajectory.

Dimension 3: Proficiency Development Through Instructional Practice directly addresses Research Question 3: Which instructional practices promote the most beneficial conceptual understanding of fractions in upper elementary classrooms? This dimension integrates the previous two by articulating how comprehensive fractions proficiency including fraction division proficiency develops across multiple competencies (Tsai & Li, 2017). This dimension encompasses not only conceptual and procedural dimensions but also how instructional practices, teacher knowledge, and assessment approaches facilitate growth (Tsai & Li, 2017). Research on children's fraction knowledge construction reveals that students build fractional understanding through progressive schemes that evolve from partitioning activities to increasingly sophisticated reasoning about fractional quantities including the partitioning and iterating activities that are most directly foundational for understanding fraction division conceptually (Steffe, 2001). Understanding these developmental progressions is essential for designing instruction that appropriately scaffolds students' movement through increasingly complex fractional concepts. Research indicates that sustainable mathematical development requires intentional pedagogical design that promotes conceptual thinking (Kazemi & Stipek, 2001), teacher beliefs aligned with conceptual learning (Stohlmann et al., 2015), and differentiated pathways supporting diverse learners. Hallett and colleagues (2010) found significant individual differences in how students develop conceptual and procedural knowledge when learning fractions, highlighting the need for differentiated instructional approaches that recognize varied developmental pathways while maintaining a conceptual foundation. Conceptual explanations specifically enhance students' ability to compare and reason with fractions (Geller et al., 2017), reinforcing how instructional practices directly impact proficiency development. Additionally, understanding learning processes supports the design of instruction that cultivates both conceptual understanding and procedural competence (Geary et al., 2008).

As illustrated in Figure 1, the intersection of these three dimensions creates a unified conceptual model: conceptual understanding of fraction division and fraction concepts broadly provides the foundation for meaningful proficiency development, which predicts sustained mathematical achievement and transfer of knowledge. The visual representation demonstrates how each dimension contributes distinct but complementary dimensions; foundation, prediction, and integration that converge to inform pedagogical practice. This unified conceptual model guides the synthesis of cognitive, instructional, and experimental research to shed light on pedagogical practices that solidify students' fraction understanding and long-term mathematical growth in upper elementary classrooms. The model depicted in Figure 1 further illustrates how these dimensions translate into three areas of application: instructional sequencing, student outcomes, and teacher practices, each of which will be explored in depth throughout this literature review.



Note. This Unified Conceptual Model synthesizes cognitive, instructional, and experimental research demonstrating how conceptual-first instruction in upper elementary fractions produces sustained mathematical growth and long-term achievement benefits.

Adapted from Bailey et al. (2012), Barkin (2019), Byrnes et al. (1991), DeWolf et al. (2015), Geary et al. (2008), Geller et al. (2017), Hallett et al. (2010), Kazemi et al. (2001), Lenz et al. (2024), Rittle-Johnson et al. (2015), Stohlmann et al. (2015), and Tsai et al. (2017).

Figure 1

Unified Conceptual Model for Conceptual-First Fraction Instruction. Three Dimensions Addressing the Research Questions.

Note. This unified conceptual model synthesizes cognitive, instructional, and experimental research demonstrating how conceptual-first instruction in upper elementary fractions produces sustained mathematical growth and long-term achievement benefits. Adapted from Bailey et al. (2012), Barkin (2019), Byrnes et al. (1991), DeWolf et al. (2015), Geary et al. (2008), Geller et al. (2017), Hallett et al. (2010), Kazemi et al. (2001), Lenz et al. (2024), Rittle-Johnson et al. (2015), Siegler et al. (2015), Stohlmann et al. (2015), and Tsai & Li, (2017).

The Unified Conceptual Model and Its Implications

The unified conceptual model presented in Figure 1 provides the organizational framework for this literature review, with fraction division serving as the specific focal concept through which the broader argument is most clearly examined and applied. The model's three dimensions map directly onto the three research questions guiding this review and converge on a single central argument: conceptual-first instruction is not merely one approach among many, but an essential pedagogical sequence grounded in how students learn, what predicts their mathematical success, and how fraction proficiency develops. This argument is most powerfully demonstrated through fraction division, where the consequences of teaching the invert-and-multiply procedure before students understand what division by a fraction means are most visible and most consequential for students' long-term mathematical trajectories.

At the foundation, Dimension 1 (Conceptual vs. Procedural Knowledge) establishes that deep relational understanding of fraction concepts must precede mechanical procedural skill development, responding to Research Question 1. When procedural instruction precedes conceptual grounding in fraction division, students develop fragmented understanding that leads to errors and limited transfer of knowledge; most visibly when students can execute the algorithm but cannot explain what their quotient represents or why dividing by a fraction less than one produces a larger result (Lenz et al., 2024). Building on this foundation, Dimension 2 (Mathematics Achievement Prediction) responds to Research Question 2 by demonstrating why this instructional sequencing produces lasting effects: early conceptual understanding of fractions, including fraction division, strongly predicts later success in ratios, proportional reasoning, algebra, and advanced mathematics (Bailey et al., 2012; DeWolf et al., 2015). Dimension 3 (Proficiency Development Through Instructional Practice) completes the model by responding to Research Question 3, articulating how comprehensive fraction proficiency develops through intentional instructional design that promotes conceptual thinking (Kazemi & Stipek, 2001), teacher beliefs aligned with conceptual learning (Stohlmann et al., 2015), and differentiated pathways that recognize individual variation in how students develop both conceptual and procedural fraction knowledge (Hallett et al., 2010).

As illustrated in Figure 1, these three dimensions translate into three areas of practical application: instructional sequencing, student outcomes, and teacher practices. When teachers prioritize conceptual understanding of fraction division first, they equip students with the cognitive structures needed to connect, apply, and extend their fraction knowledge over time (Geller et al., 2017; Stohlmann et al., 2015), cultivating the ability to transfer knowledge to more advanced mathematical topics (DeWolf et al., 2015), and reinforcing ongoing proficiency development through carefully designed instructional practice (Tsai & Li, 2017). The subsequent sections of this review systematically examine the research supporting each dimension and their integration, illustrating how conceptual-first fraction division instruction fosters both immediate understanding and long-term mathematical success in upper elementary classrooms.

Conceptual vs. Procedural Knowledge in Fraction Learning

A core distinction in mathematics education research lies between conceptual and procedural knowledge, understanding this distinction in both general and specific terms is essential before examining its instructional implications. In general, conceptual knowledge refers to a deep understanding of mathematical ideas, principles, and the relationships between them, while procedural knowledge involves knowing how to carry out specific computational steps in a reliable and accurate sequence (Schneider et al., 2011). Conceptual knowledge is relational; it allows a learner to understand why a mathematical process works and to connect that process to other ideas. Procedural knowledge is sequential; it allows a learner to execute a series of steps to arrive at a correct answer (Schneider et al., 2011). Neither form of knowledge is trivial, both are necessary for full mathematical competence. However, research consistently demonstrates that the order in which they are developed matters significantly for long-term understanding (Rittle-Johnson et al., 2001).

When applied specifically to fraction division, this distinction becomes both concrete and consequential. Procedural knowledge of fraction division centers on the invert and multiply algorithm: to divide by a fraction, a student rewrites the problem by keeping the first fraction, changing the operation to multiplication, and flipping the second fraction. For example, $\frac{3}{4} \div \frac{1}{2}$ becomes $\frac{3}{4} \times \frac{2}{1}$, yielding $\frac{6}{4}$ or $1\frac{1}{2}$. A student with procedural knowledge can execute these steps accurately and efficiently. They know to identify the divisor, find its reciprocal, and multiply across the numerators and denominators. They can also apply related procedural steps such as simplifying the resulting fraction or converting between improper fractions and mixed numbers (Barkin, 2019). These are not insignificant skills, and students who develop procedural fluency can perform fraction division correctly under timed or routine conditions (Lenz et al., 2024).

Conceptual understanding of fraction division, however, demands considerably more. A student with conceptual understanding can explain what the expression $\frac{3}{4} \div \frac{1}{2}$ actually means: how many groups of $\frac{1}{2}$ are contained in $\frac{3}{4}$. They can represent this question using a visual model, such as drawing a number line or an area diagram, and use that representation to justify why the answer is $1\frac{1}{2}$. They understand that dividing by a fraction less than one produces a quotient larger than the original, and they can explain why this is true rather than simply accepting it as a quirk of the algorithm. They can also connect fraction division to real-world contexts, such as determining how many half-cup servings are in three-quarters of a cup of rice. Rešić and colleagues (2016) argued that effective fraction instruction requires a deliberate progression from concrete representations through pictorial models to abstract symbolic manipulation, and it is this progression that characterizes genuine conceptual understanding. Historical reviews of fraction research demonstrate that instruction has traditionally emphasized procedural algorithms while neglecting these conceptual foundations, contributing to persistent and widespread learning difficulties across grade levels (Pitkethly & Hunting, 1996).

Addressing Research Question 1: What are the effects of conceptually teaching fractions of students' long-term mathematical understanding compared to procedural instruction? Dimension 1 of the Unified Conceptual Model examines this foundational distinction and its instructional implications for fraction division specifically. This dimension advances a clear instructional position: conceptual understanding must be established before procedural instruction if students are to develop meaningful, lasting mathematical knowledge of fraction division and the broader mathematical concepts it supports.

The relationship between conceptual and procedural knowledge in fraction learning has been studied extensively over several decades. Byrnes and Wasik (1991) established early evidence that conceptual knowledge facilitates procedural learning, demonstrating that students with stronger conceptual foundations acquire procedures more readily and with greater accuracy. Their foundational work revealed that the role of conceptual knowledge extends beyond mere understanding; it actively supports the development of procedural skills. This is especially significant in fraction division, where the algorithm is notoriously counterintuitive. Students who have first built a conceptual understanding of what division by a fraction means are far better positioned to make sense of why inverting and multiplying is mathematically valid than students who encounter the algorithm first and are left to accept it on faith. Subsequent research confirmed that the relationship between conceptual and procedural knowledge operates as an iterative process, wherein gains in one domain lead to gains in the other, creating a reciprocal cycle of mathematical development (Rittle-Johnson et al., 2001). This iterative framework suggests that instruction should strategically leverage this bidirectional relationship by establishing strong conceptual foundations that initiate and sustain the developmental cycle. More recent research by Rittle-Johnson and colleagues (2015) expanded this understanding by confirming that the relationship is bidirectional, but their findings still emphasize that initial conceptual grounding provides the strongest foundation for this reciprocal development, particularly in early stages of learning complex topics like fraction division.

Understanding how children construct fractional knowledge provides critical insights for instructional design. Steffe (2001) proposed that children's fractional knowledge develops through a progression of schemes, beginning with partitioning activities that allow them to create equal parts and progressing toward more sophisticated understanding of fractions as numbers in their own right. In the context of fraction division, this means students need extensive experience partitioning quantities and reasoning about how many equal groups fit within a given amount before they are ready to represent that relationship symbolically. Effective instruction must account for these developmental progressions, providing students with appropriately sequenced experiences that support the construction of increasingly sophisticated fractional schemes rather than bypassing them through early algorithmic instruction.

A comprehensive review of fraction learning research across global contexts spanning 35 years reveals consistent patterns in student difficulties and effective instructional approaches (Syed Ismail et al., 2024). The bibliometric analysis identified that successful fraction instruction requires multiple representational systems, emphasis on conceptual understanding before procedural execution, and explicit attention to common misconceptions that emerge when students overgeneralize whole number reasoning to fractional contexts. In fraction division specifically, this overgeneralization is particularly damaging: students who reason from whole number logic frequently expect that division should always make a quantity smaller, and without conceptual grounding they have no framework for understanding why dividing by $\frac{1}{2}$ produces a larger result. These findings reinforce that challenges in fraction division are not isolated to specific educational contexts but represent fundamental cognitive and instructional issues requiring systematic attention to conceptual development.

Research consistently shows that the sequencing of these two types of knowledge acquisition directly affects students' mathematical development in fraction division and beyond. When procedural instruction precedes conceptual understanding, students often develop fragmented understandings that hinder problem solving and retention (Lenz et al., 2024). Siegler and Lortie-Forgues (2015) investigated conceptual knowledge of fraction arithmetic and found that students' conceptual understanding of why fraction algorithms work was more strongly associated with problem-solving success than their ability to execute procedures alone. This finding is particularly relevant to fraction division, where the invert-and-multiply procedure is one of the most commonly memorized but least understood algorithms in elementary mathematics. Students who understood the conceptual basis for the operation were better able to apply their knowledge flexibly across varied problem contexts, while those who had only procedural knowledge struggled when problems were presented in unfamiliar formats or real-world contexts. When conceptual understanding is emphasized first, it creates a cognitive foundation that enables students to connect symbolic representations with meaning, leading to more flexible and accurate procedural application (Barkin, 2019). Conceptual-first instruction helps students build a schema that allows them to reason through unfamiliar problems rather than rely solely on memorized algorithms.

Wiest and Amankonag (2019) compared conceptual and procedural approaches to fraction tasks and found that students using conceptual strategies demonstrated deeper understanding and were better able to explain their reasoning compared to those relying on procedural algorithms. Their review revealed that students taught conceptually could flexibly apply multiple strategies, such as using visual models or reasoning about the relationship between numerators and denominators, while procedurally trained students were limited to executing steps without understanding why they worked. This research provides concrete evidence that conceptual approaches yield not only deeper understanding but also greater strategic flexibility, precisely the kind of flexibility students need when fraction division concepts extend into ratios, proportional reasoning, and algebra in later grades.

Individual learners may develop conceptual and procedural knowledge through different pathways and at different rates. Hallett and colleagues (2010) found significant individual differences in how students develop these knowledge types when learning fractions, with some students showing stronger initial conceptual understanding while others demonstrated procedural facility first. Importantly, their research revealed that students who began with stronger conceptual knowledge showed greater long-term gains in both conceptual and procedural domains, highlighting the value of establishing conceptual foundations early. These findings underscore the need for differentiated instructional approaches that recognize varied developmental pathways while maintaining an overall emphasis on conceptual foundation as the optimal starting point for fraction division instruction.

Effective classroom practices that promote conceptual reasoning in fraction division include the use of multiple representations, discussion-based instruction, and real-world problem contexts (Kazemi & Stipek, 2001). Teachers who encourage students to justify their reasoning using visual models, compare solution strategies, and connect the invert-and-multiply algorithm to what it means to find how many groups of one fraction fit into another promote richer mathematical discourse that deepens understanding. This approach contrasts sharply with procedural drill, which often emphasizes speed and accuracy over comprehension and leaves students unable to explain or apply what they have learned. The cumulative evidence across these studies indicates that conceptual instruction in fraction division not only enhances immediate understanding but also produces long-term growth in mathematical reasoning and flexibility that supports students' success in the more advanced mathematical domains that follow.

Conceptual Understanding as a Predictor of Long-Term Mathematical Achievement

Conceptual understanding of fractions and fraction division specifically extends beyond immediate computational competence. It serves as a powerful predictor of future mathematical achievement. This section addresses Research Question 2: How does conceptual understanding of fractions aid students' ability to transfer knowledge to later concepts such as ratios, proportional reasoning, and algebra? The following research supports Dimension 2 of the Unified Conceptual Model. Bailey, Hoard, Nugent, and Geary (2012) found that early proficiency with fractions strongly predicts later success, as fractions represent a critical cognitive bridge between arithmetic and higher-level mathematical concepts. Fraction division is particularly significant within this predictive relationship because it requires students to reason about quantities in ways that directly mirror the relational thinking demanded by ratios, proportional reasoning, and algebraic expressions. Students who develop conceptual fraction knowledge are better equipped to understand ratios, proportions, and algebraic relationships because they have internalized the underlying structures of rational numbers.

DeWolf, Bassok, and Holyoak (2015) demonstrated that students with strong conceptual foundations in fractions exhibit greater relational reasoning abilities when comparing or manipulating rational numbers. Their extended literature review revealed that understanding the conceptual structure of fractions enables learners to transfer knowledge to decimals and algebraic contexts, underscoring the role of conceptual thinking in promoting cognitive flexibility. Students who understand fraction division conceptually, who can explain what it means to divide by a fraction and why the quotient behaves as it does, are better positioned to recognize the same relational structure when it appears in ratio and proportional reasoning contexts in later grades. Similarly, the National Mathematics Advisory Panel (Geary et al., 2008) identified fraction competence as one of the most important predictors of later mathematical success, emphasizing that meaningful learning in early mathematics provides a foundation for more abstract reasoning.

Taken together, these findings illustrate that conceptual understanding is not only vital for mastering fraction operations but also for supporting the transfer of knowledge across mathematical domains. Conceptually grounded instruction enables students to perceive connections among mathematical ideas, allowing them to approach new topics with a coherent understanding rather than isolated skills. This evidence supports the argument that teaching fractions conceptually first has lasting benefits that extend well beyond elementary mathematics.

Developing Fraction Proficiency Through Instructional Practice

Developing authentic fraction proficiency requires the integration of conceptual, procedural, and adaptive knowledge. This section addresses Research Question 3: Which instructional practices promote the most beneficial conceptual understanding of fractions in upper elementary classrooms? Some of the answers are found in the research that supports Dimension 3 of the Unified Conceptual Model. Tsai and Li (2017) developed a framework specifically focused on fraction proficiency across upper elementary grades, proposed a comprehensive model that highlights how students progress through interconnected dimensions of understanding, computation, and application of fraction concepts. This model stresses that deep learning of fractions occurs when conceptual knowledge supports procedural fluency, not when the two are taught in isolation.

Instructional practices play a decisive role in fostering conceptual understanding of fractions. Research by Kazemi and Stipek (2001), conducted in upper elementary mathematics classrooms examining how classroom discourse affects mathematical reasoning across topics including fractions, found that classrooms emphasizing mathematical discussion, reasoning, and student exploration promote stronger conceptual gains than those focused primarily on procedural execution. Geller, Son, and Stigler (2017), studying students’ understanding of fraction comparisons specifically, found that when students are asked to provide conceptual explanations for how they determined which fraction was greater, they demonstrate a more durable and transferable understanding than students who simply calculated answers. These practices help students connect symbolic fraction representations with meaningful quantities and encourage flexible problem solving across fraction contexts.

Contemporary instructional tools can also support conceptual development of fraction concepts when thoughtfully designed. Zhang and colleagues (2020), examining the use of digital games specifically designed to build conceptual knowledge of fractions in primary grade students, demonstrated that these tools significantly supported learning when games emphasized visual representations of fraction magnitude and problem solving over procedural drill. Their review found that students using conceptually focused digital games showed greater improvement in understanding fraction magnitude, equivalence, and comparison than students using traditional worksheets focused on procedural practice. This research suggests that technology, when intentionally designed around conceptual principles specific to fraction learning can complement and enhance traditional conceptual instruction by providing interactive, engaging formats for exploring fractional relationships.

Teacher beliefs and instructional decisions also shape how conceptual understanding of fraction division and other fraction concepts develops in the classroom. A critical factor in implementing conceptual fraction instruction is teachers’ own conceptual understanding of fraction operations they teach. Copur-Gencturk (2021), conducting a national survey of elementary teachers focused specifically on their conceptual understanding of fraction operations including division, multiplication, and equivalence, found significant gaps in understanding, revealing that many educators lack the deep conceptual knowledge necessary to teach fractions meaningfully. Her research showed that teachers with stronger conceptual understanding were more likely to provide conceptual explanations to students, ask probing questions that elicited student reasoning about why fraction procedures work, and recognize and address student misconceptions about fraction concepts. Conversely, teachers with primarily procedural knowledge of fractions tended to focus instruction on algorithmic execution, such as the invert-and-multiply procedure for division, without connecting those procedures to underlying meaning. This finding underscores the critical importance of professional development that strengthens teachers' own conceptual understanding of fraction operations before they can effectively implement conceptual-first instruction in their classrooms.

Similarly, Reeder and Utley (2017), examining prospective elementary teachers' understanding of fraction concepts specifically, found that many struggled to articulate what a fraction is beyond procedural definitions such as "a number with a numerator and denominator" or "a division problem." When asked to explain fundamental fraction concepts such as why multiplying by a fraction less than one yields a smaller product, or why dividing by a fraction produces a larger quotient, many pre-service teachers could not provide conceptual justifications and instead restated the procedures. This research highlights the need for teacher preparation programs to prioritize conceptual development of fraction content knowledge specifically, ensuring that future teachers can model and facilitate conceptual thinking about fractions operations for their students.

Stohlmann and colleagues (2015), studying pre-service teachers' beliefs about mathematics instruction broadly, found that those who shifted their beliefs toward viewing mathematics as a conceptual, sense-making practice rather than a set of rules to be memorized were more likely to engage students in meaningful learning experiences. While this research addresses teacher beliefs about mathematics generally, its implications are particularly significant for fraction instruction, where the temptation to rely on memorable tricks and mnemonics such as “leave it, change it, flip it” is especially strong and especially limiting for students’ long-term understanding. Effective professional development and teacher preparation that address both general mathematical beliefs and fraction specific conceptual knowledge are therefore essential components of promoting conceptual fraction instruction in classrooms.

Cognitive science research further supports these instructional findings. Geary and colleagues (2008), drawing on a broad review of mathematics learning research across number domains including fractions, highlighted that learning processes must align with how children construct mathematical meaning. In the context of fraction division specifically, this means instructional designs must incorporate scaffolding that moves students from concrete representations of dividing quantities into equal groups, through pictorial models, to the abstract symbolic algorithm, with feedback and opportunities for reflection at each stage. These experiences help students build mental connections between what fraction division means and how the invert-and-multiply procedure enacts that meaning. Collectively, these studies illustrate that developing fraction proficiency, and fraction division proficiency in particular, requires intentional instructional design that begins with conceptual understanding and evolves into procedural mastery through carefully sequenced practice and application.

Application to Fifth and Sixth Grade Classrooms

The breadth of fraction concepts encountered in fifth and sixth grade; including improper fractions, multiple interpretations of fractions as quotients, ratios, operators, and measures, and the partitioning understanding that underlies all fraction reasoning makes it necessary to identify which specific fraction concept this review focuses on most closely. These topics are not equally weighted in the research literature, nor are they equally consequential for students’ long-term mathematical trajectories. This review focuses specifically on fraction division as its primary fraction concept of examination, for several reasons. Fraction division is the operation that most completely draws on and requires all of the conceptual foundations that the preceding research has established as critical: students must understand what a fraction represents as a quantity, how partitioning creates equal groups, what the quotient interpretation means, and why dividing by a value less than one produces a larger result. It is also the operation most commonly taught through a memorized procedure with little to no conceptual grounding, making it the clearest and most consequential example of what is lost when procedural instruction precedes conceptual understanding (Reeder & Utley, 2017; Copur-Gencturk, 2021). For these reasons, fraction division serves throughout this review as the specific lens through which the broader argument for conceptual-first instruction is examined and applied.

The research on conceptual fraction instruction has particular significance for fifth and sixth grade classrooms, where students encounter increasingly complex fraction concepts including fraction division, improper fractions, and the transition toward ratio and proportional reasoning. At this developmental stage, students are cognitively ready to move beyond basic part-whole interpretations toward more sophisticated understandings of fractions as numbers, operators, quotients, ratios, and measures (Pitkethly & Hunting, 1996). However, this transition is successful only when students have developed strong conceptual foundations in earlier grades or receive intentional instruction designed to build those foundations, particularly around the meaning of fraction operations such as division before the algorithms for those operations are introduced.

Research specifically examining how children construct understanding of improper fractions demonstrates the critical role teachers play in promoting conceptual understanding during fifth and sixth grade. Tzur (1999) conducted a detailed study of how upper elementary students build conceptual schemes for improper fractions, found that teacher facilitation significantly impacts whether students develop meaningful concepts or resort to procedural manipulation. This research revealed that when teachers engage students in partitioning activities and reflective abstraction; asking students to think about their thinking as they create and reason about quantities greater than one whole, students successfully construct schemes for understanding improper fractions as genuine quantities rather than as symbols to be converted. Conversely, when instruction focused primarily on converting improper fractions to mixed numbers through algorithmic procedures without conceptual grounding, students demonstrated fragmented understanding and could not flexibly reason about these quantities in problem-solving contexts. This finding is directly relevant to fraction division in fifth and sixth grade, where students who do not understand improper fractions conceptually will struggle to interpret quotients that are greater than one whole.

The teacher's role extends beyond simply presenting conceptual models; it involves carefully orchestrating learning experiences that allow students to construct increasingly sophisticated fractional schemes. Tzur (1999) emphasized that teachers must understand the progressive nature of children's fraction knowledge construction, recognizing that students build understanding through stages that cannot be rushed through procedural shortcuts. In fifth and sixth grade classrooms, this means providing experiences where students physically partition quantities, represent fractions using multiple models such as area models, number lines, and set models, and engage in mathematical discourse that makes their reasoning visible. For fraction division specifically, this looks like asking students to draw what $\frac{3}{4} \div \frac{1}{2}$ means before they are ever shown the invert-and-multiply procedure, so that the algorithm becomes an efficient shortcut for something they already understand rather than an arbitrary rule. Teachers who understand Steffe’s (2001) theoretical framework describing how children’s fractional knowledge develops through progressive schemes can design instruction that supports students’ progression from partitioning activities through to understanding fractions as composite units and ultimately as numbers in their own right.

The iterative relationship between conceptual and procedural knowledge has specific implications for upper elementary fraction instruction. Rittle-Johnson and colleagues (2001), studying how conceptual and procedural knowledge develop together in mathematics learning broadly, found that students benefit most when instruction alternates strategically between conceptual exploration and procedural practice, with each phase building upon and reinforcing the other. For fifth and sixth grade teachers working with fraction division, this suggests that lessons should cycle between hands-on conceptual activities such as using fraction bars or number lines to explore what it means to divide one fraction by another, and opportunities to practice the invert-and-multiply procedure with understanding, maintaining visual reference to what the algorithm is actually doing. This iterative approach prevents the common pattern where students learn the procedure for fraction division in isolation and subsequently struggle to apply it meaningfully or interpret what their answer represents.

Contemporary research across global contexts provides additional guidance for fifth and sixth grade fraction instruction. The bibliometric analysis by Syed Ismail et al. (2024), synthesizing 35 years of fraction learning research across international educational contexts, identified consistent themes particularly relevant to upper elementary classrooms: the necessity of multiple representational systems, the importance of explicit attention to common misconceptions, and the value of connecting fraction concepts to real-world contexts. Fifth and sixth grade students commonly overgeneralize whole number reasoning to fraction division, expecting for example that division should always make a quantity smaller, and instruction must explicitly address this misconception through conceptual experiences that create cognitive conflict and resolution before the algorithm is introduced.

For teachers implementing conceptual-first instruction in grades five and six, the research provides clear guidance. Pitkethly and Hunting (1996), reviewing decades of research on how students develop understanding across multiple interpretations of fractions, emphasized that students need extensive experience with all fraction interpretations; not just part-whole relationships but also quotient, ratio, operator, and measure interpretations. This is especially important for fraction division, which draws on the quotient interpretation of fractions and requires students to understand division as finding how many equal groups of one quantity are contained in another. Upper elementary students who engage with fractions through multiple interpretive lenses develop more flexible understanding that supports later work with ratios, proportions, and algebra. Additionally, the developmental progressions identified by Steffe (2001) in his theoretical framework for children's fractional knowledge suggest that fifth and sixth grade instruction should include explicit opportunities for students to engage in partitioning activities, even if these seem elementary, as they provide the foundational schemes necessary for understanding fraction division conceptually rather than procedurally.

The practical application of this research in fifth and sixth grade classrooms requires teachers who possess deep conceptual understanding of fraction operations, including fraction division, themselves (Copur-Gencturk, 2021) and who understand the developmental trajectories through which students construct fractional knowledge (Steffe, 2001; Tzur, 1999). When teachers combine this knowledge with instructional practices that emphasize conceptual reasoning before procedural execution; ensuring students can represent and explain fraction division before they are asked to execute the algorithm, they create optimal conditions for students to develop both immediate fraction proficiency and the conceptual foundations that will support their success in algebra and advanced mathematics.

The Critical Role of Teacher Preparation in Fraction Instruction

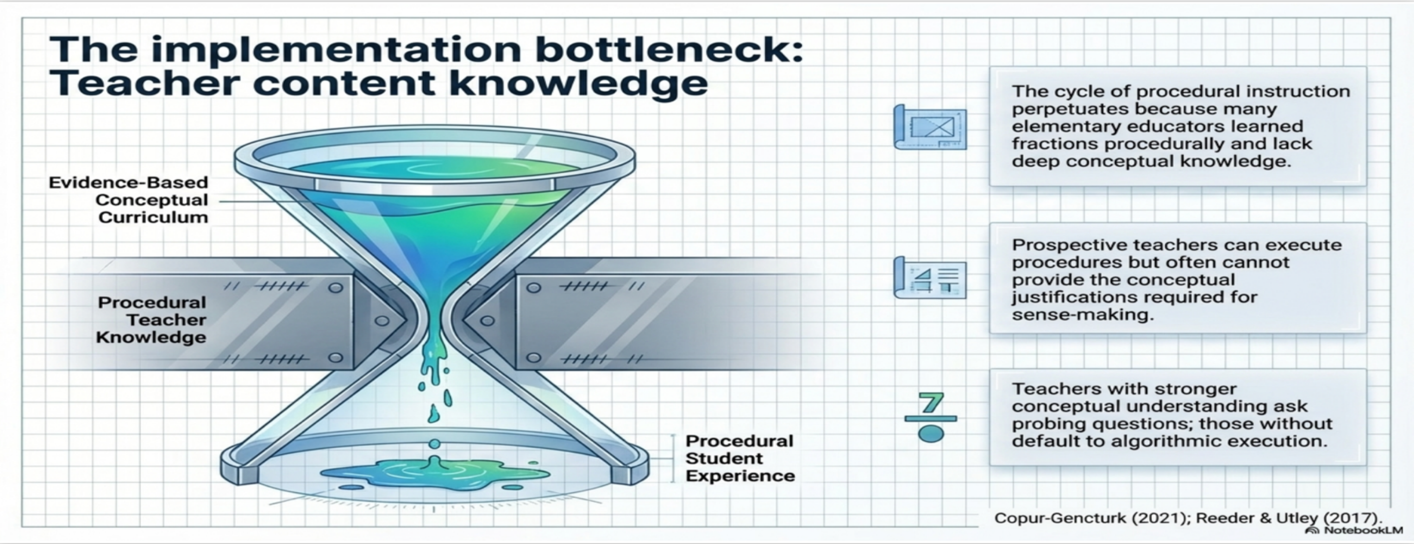
The research evidence overwhelmingly demonstrates that successful implementation of conceptual fraction instruction depends fundamentally on teachers' own conceptual understanding and pedagogical preparation. The cycle of procedural-focused instruction perpetuates across generations not because of curricular deficiencies alone, but because many teachers themselves learned fractions procedurally and lack the deep conceptual knowledge necessary to teach differently (Copur-Gencturk, 2021; Reeder & Utley, 2017). This creates a critical imperative for teacher preparation programs and professional development to prioritize the development of both mathematical content knowledge and pedagogical content knowledge specific to fractions.

Research reveals significant gaps in prospective teachers' fraction understanding that directly impact their future instructional capacity. Reeder and Utley (2017) found that many preservice elementary teachers struggled to articulate what a fraction is beyond superficial procedural definitions such as "a number with a numerator and denominator" or "a division problem." When asked to explain fundamental concepts such as why multiplying by a fraction less than one yields a smaller product, or why the procedure for dividing fractions involves "flipping" the second fraction, many could not provide conceptual justifications. This research reveals a troubling pattern: prospective teachers can execute fraction procedures but cannot explain why these procedures work, which severely limits their ability to provide the conceptual explanations that research shows are essential for student learning (Geller et al., 2017).

The importance of teachers' conceptual understanding extends beyond their ability to explain procedures; it fundamentally shapes the nature of classroom instruction. Copur-Gencturk (2021) conducted a national survey of elementary teachers and documented substantial gaps in teachers' conceptual understanding of fraction operations. Her research demonstrated that teachers with stronger conceptual understanding engaged in qualitatively different instructional practices: they provided conceptual explanations rather than just procedural steps, asked probing questions that elicited student reasoning, used multiple representations flexibly, and recognized and addressed student misconceptions more effectively. Conversely, teachers with primarily procedural knowledge tended to focus instruction narrowly on algorithmic execution, relied heavily on mnemonic devices and tricks, and struggled to respond to students' conceptual questions. This finding underscores that teacher knowledge is not merely a background qualification but an active determinant of instructional quality. Figure 2 illustrates how this bottleneck operates in practice: even when evidence-based conceptual curriculum exists, procedural teacher knowledge constrains what reaches students, resulting in procedural student experience rather than the conceptual understanding the curriculum was designed to develop.

Figure 2

The Implementation Bottleneck: Teacher Content Knowledge



The implementation bottleneck: Teacher content knowledge

The cycle of procedural instruction perpetuates because many elementary educators learned fractions procedurally and lack deep conceptual knowledge.

Prospective teachers can execute procedures but often cannot provide the conceptual justifications required for sense-making.

Teachers with stronger conceptual understanding ask probing questions; those without default to algorithmic execution.

Copur-Gencturk (2021); Reeder & Utley (2017).

Note: This figure illustrates how procedural teacher knowledge functions as a bottleneck between evidence-based conceptual curriculum and student mathematical experience. Teachers who learned fractions procedurally and lack deep conceptual knowledge are unable to fully implement conceptual instruction, resulting in students receiving procedural rather than conceptual learning experiences. Adapted from Copur-Gencturk (2021) and Reeder & Utley (2017). *"With visual design generated using NotebookLM"*

Understanding how children construct fractional knowledge is equally essential for teacher preparation. Tzur's (1999) research on improper fractions revealed that teachers who understood students' developmental progressions were able to design learning experiences that promoted genuine conceptual construction, while teachers lacking this knowledge defaulted to procedural instruction even when attempting to teach conceptually. The extended literature review demonstrated that effective teaching requires teachers to understand not only the mathematical content but also how students construct fractional schemes through progressive stages, from initial partitioning activities through increasingly sophisticated reasoning about fractional quantities (Steffe, 2001) and that instruction must be carefully sequenced to support this construction rather than attempting to shortcut it through procedural memorization.

Teacher preparation must also address the iterative nature of conceptual and procedural knowledge development. Rittle-Johnson et al. (2001) found that both forms of knowledge develop in tandem through reciprocal influence, but many teachers lack understanding of how to leverage this relationship instructionally. Effective teacher preparation would help prospective teachers understand that conceptual and procedural knowledge are not separate curricular units to be taught sequentially, but interconnected forms of understanding that develop through carefully designed instructional cycles. Teachers need preparation in designing lessons that strategically alternate between conceptual exploration and procedural practice, with each phase building upon and reinforcing the other.

Historical reviews of fraction instruction reveal that many of the challenges students face today are the same challenges identified decades ago, suggesting that changes in curriculum alone are insufficient without corresponding changes in teacher preparation (Pitkethly & Hunting, 1996). Contemporary research across global contexts confirms this pattern: despite widespread recognition of fractions as a critical mathematical topic, instructional practices in many classrooms remain heavily procedural, and student difficulties persist (Syed Ismail et al., 2024). Breaking this cycle requires systematic attention to teacher preparation that develops both conceptual mathematical knowledge and the pedagogical knowledge necessary for effective fraction instruction.

Effective teacher preparation programs must therefore include several key components. First, they must provide prospective teachers with opportunities to develop their own conceptual understanding of fractions through the same kinds of experiences they will eventually provide to students, working with multiple representations, exploring multiple interpretations of fractions, and engaging in mathematical discourse that makes reasoning visible (Pitkethly & Hunting, 1996). Second, they must explicitly address common misconceptions that both teachers and students hold, helping prospective teachers understand not only the correct concepts but also knowledge of developmental progressions in students' fraction learning, so teachers can recognize where students are in their conceptual development and design instruction accordingly (Steffe, 2001; Tzur, 1999). Finally, they must help teachers understand the bidirectional relationship between conceptual and procedural knowledge and how to design instruction that leverages this relationship effectively (Rittle-Johnson et al., 2001).

The evidence consistently demonstrates that teacher preparation is not merely a supportive factor but a necessary condition for implementing conceptual-first fraction instruction successfully. Without teachers who possess deep conceptual understanding of fractions, knowledge of how students construct fractional concepts, and pedagogical skills to facilitate this construction, even the most well-designed curricula will be implemented in ways that perpetuate procedural emphasis (Copur-Gencturk, 2021). Consequently, efforts to improve fraction instruction must address teacher preparation as a central priority, ensuring that future teachers enter classrooms equipped with the knowledge and skills necessary to break the cycle of procedural instruction and provide students with the conceptual foundations they need for long-term mathematical success.

Counterpoints and Alternative Perspectives on Conceptual-First Fraction Instruction

While the literature reviewed thus far presents compelling evidence for conceptual-first instruction in fractions, recent research challenges several foundational assumptions underlying this approach. Four studies in particular raise important questions about the timing, effectiveness, and underlying mechanisms of conceptual instruction in fraction learning specifically, suggesting that the relationship between conceptual and procedural knowledge in fraction contexts may be more complex than previously theorized. Importantly, these counterpoints do not dismantle the case for conceptual-first instruction, rather they refine and strengthen it by revealing that procedural instruction alone is demonstrably insufficient, and that the most effective path forward begins with conceptual grounding while remaining responsive to how individual students develop both forms of knowledge over time.

The Persistence of Whole Number Bias Despite Conceptual Instruction

Braithwaite and Siegler (2018), conducting a longitudinal study tracking fourth through eighth grade students' understanding of fraction magnitude specifically, present evidence that fundamentally challenges optimistic assumptions about conceptual instruction's ability to prevent cognitive biases in fraction learning. Their research documents that whole number bias; the tendency to focus on separate whole number components such as numerator and denominator rather than on a fraction's integrated magnitude, persists well into middle school and even adulthood, despite years of fraction instruction. The researchers found that students treated equivalent fractions with larger components, such as $\frac{15}{20}$, as larger than those with smaller components, such as $\frac{4}{5}$, demonstrating that whole number bias decreased between fourth and eighth grade but did not disappear entirely even with ongoing fraction instruction (Braithwaite & Siegler, 2018). Critically, this bias was evident not only in students newly introduced to fractions but also in mathematicians tested under speeded conditions, suggesting the interference of whole number reasoning with fraction magnitude understanding is deeply rooted and resistant to even extensive mathematical training.

This finding directly challenges Steffe's (2001) theoretical framework about how children progressively construct fractional knowledge through partitioning schemes, and contradicts the assumption implicit in Rittle-Johnson and colleagues' (2001) iterative model that early conceptual instruction in fractions can establish cognitive foundations strong enough to prevent persistent biases. This persistence suggests that fraction learning, and fraction division in particular where students must reason about quantities that behave counterintuitively relative to whole number logic, is more cognitively complex than simple conceptual change models predict. It raises important questions about whether conceptual-first instruction alone can overcome deeply rooted whole number knowledge structures when students encounter fraction division.

The implications for the theoretical frameworks presented in this review are significant. If whole number bias persists despite conceptual fraction instruction and even among individuals with extensive mathematical training, then the Conceptual vs Procedural Knowledge Framework may overestimate the power of early conceptual foundation to shape long-term understanding of fraction concepts. The research suggests that cognitive biases from whole number knowledge interfere with fraction understanding in ways that resist instructional intervention, challenging the notion that proper conceptual instruction early in fraction learning can fully prevent such difficulties (Braithwaite & Siegler, 2018). What this finding calls for is not abandoning conceptual instruction, but rather making it more explicit, more sustained, and more responsive to students' ongoing reasoning as they encounter these biases throughout their fraction learning.

Uncertainty About the Origins and Solutions for Whole Number Bias

Building directly on this challenge, Ni and Zhou (2005), conducting a comprehensive theoretical review of research on whole number bias in fraction learning specifically, reveal that the problem runs even deeper than Braithwaite and Siegler's findings suggest. Not only does whole number bias persist despite conceptual fraction instruction, but researchers cannot yet agree on why it exists or how to effectively address it. Their review of three competing explanations for whole number bias in fraction contexts reveals fundamental disagreements about the cognitive mechanisms underlying fraction learning difficulties. The authors argue that researchers disagree on the origin of whole number bias in fraction learning, with accounts diverging on whether early quantitative representation originates from a numerical-specific cognitive mechanism and whether early quantitative representations privilege discrete quantity over the continuous quantities that fractions represent (Ni & Zhou, 2005). Most critically, their analysis of the fraction learning research concludes that there does not yet appear to be sufficient evidence to decide among the competing accounts as to the nature of whole number bias in fraction contexts (Ni & Zhou, 2005).

This theoretical uncertainty has direct implications for fraction instructional design. If researcher cannot agree on the fundamental cognitive mechanisms underlying fraction learning difficulties, then prescriptive claims about optimal sequencing of conceptual and procedural fraction instruction rest on incomplete theoretical foundations. Ni and Zhou (2005) raise two essential questions that challenge conceptual-first approaches to fraction instruction specifically: How might fraction learning be organized to take advantage of prior whole number knowledge rather than having it interfere? What causes the gap between learning fraction number symbols and learning fraction number concepts? These questions suggest that the problem of teaching fractions may not be simply a matter of teaching fraction concepts before fraction procedures, but rather involves more fundamental issues about how new fractional understanding builds upon or conflicts with existing whole number knowledge structures.

The theoretical ambiguity identified by Ni and Zhou (2005) challenges the confident assertions in the frameworks presented earlier in this review regarding fraction instruction. If the cognitive basis for whole number bias in fraction learning remains contested, then claims about how conceptual understanding of fractions prevents or mitigates these biases may be premature. Their analysis indicates that fraction learning difficulties may be more intractable than instructional reform of sequencing alone can address, and that fraction division in particular where whole number intuitions about division making quantities smaller are directly contradicted, may require more targeted instructional attention than a general conceptual-first approach provides. Taken together, Braithwaite and Siegler (2018) and Ni and Zhou (2005) present a unified challenge; whole number bias in fraction learning is both persistent and poorly understood, which significantly complicates confident prescriptions about how conceptual instruction can prevent or resolve it. Yet this unified challenge also reinforces a critical point; if even conceptual instruction struggles to fully overcome these biases, procedural instruction alone leaves students entirely without recourse when these deeply rooted cognitive conflicts emerge.

Questioning the Empirical Separability of Conceptual and Procedural Knowledge

Beyond the challenge of whole number bias, a separate but related body of research questions whether the conceptual and procedural knowledge that fraction instruction aims to develop are as distinct and sequentially ordered as the frameworks in this review assume. Lenz, Dreher, Holzapfel, and Wittmann (2020), studying 235 eighth grade students in Germany specifically examining how conceptual and procedural fraction knowledge develop and relate to one another, challenge fundamental assumptions about the relationship between these two knowledge types in fraction learning. While previous research, including the iterative model proposed by Rittle-Johnson and colleagues (2001) examining mathematics learning broadly, treats conceptual and procedural knowledge as distinct yet interconnected forms of understanding, Lenz and colleagues questioned whether these constructs are empirically separable in fraction learning specifically. Their study used sophisticated confirmatory factor analysis to test whether conceptual and procedural fraction knowledge represent truly distinct dimensions in how students actually develop understanding of fraction concepts.

The researchers found that while conceptual and procedural fraction knowledge could be measured as separate constructs, individual students' knowledge profiles varied considerably and remained stable over time, with transition paths showing gradual improvement in both knowledge types rather than one type consistently preceding or catching up to the other (Lenz et al., 2020). This finding complicates the sequential model implicit in conceptual-first fraction instruction. Rather than conceptual understanding of fraction concepts serving as a clear foundation for procedural skill development, Lenz and colleagues found substantial individual variation in how students develop these knowledge types in fraction learning, with some students showing stronger initial conceptual understanding while others demonstrated procedural facility with fraction algorithms first.

These results pose a significant challenge to the Proficiency Development Framework outlined in this review, which assumes a relatively uniform developmental pathway with conceptual understanding of fractions establishing the foundation for procedural learning of fraction algorithms. If individual students follow different developmental trajectories when learning fractions; some conceptual-first, others procedural-first, and still others developing both types of knowledge in parallel, then instructional approaches emphasizing a single sequence may not serve all learners equally well. As Lenz and colleagues (2020) note, the relationship between conceptual and procedural fraction knowledge may be more individualized and complex than the iterative model suggests, potentially requiring more differentiated instructional approaches than conceptual-first fraction instruction alone can provide. Importantly, however, this finding does not undermine the value of beginning with conceptual instruction, it underscores the necessity of teachers actively listening to and assessing their students throughout fraction instruction in order to recognize where each student is in their development and respond accordingly.

Limitations of Part-Whole Approaches and the Case for Measurement Concepts

Wilkins and Norton (2018), synthesizing data from over 300 students across multiple studies examining how students develop fraction concepts, present perhaps the most direct challenge to the instructional assumptions underlying the research reviewed in this paper. Their research argues that the challenge with fraction learning may stem from the limitations of part-whole concepts of fractions, which form the central focus of fraction curriculum and instruction in the United States (Wilkins & Norton, 2018). This critique fundamentally challenges Pitkethly and Hunting's (1996) emphasis on part-whole understanding of fractions and Tzur's (1999) focus on partitioning activities as the foundational entry point for fraction learning, both of which were discussed earlier in this review as supports for conceptual-first instruction.

Wilkins and Norton (2018) propose an alternative learning progression for fraction concepts based on measurement, rather than part-whole relationships. Their research demonstrates that students need opportunities to engage in activities that support the coordination of partitioning and iterating and the coordination of three levels of units inherent in fractions, in order to develop robust fraction understanding (Wilkins & Norton, 2018). This measurement-based approach to fraction concepts suggests that part-whole understanding; far from being the essential entry point emphasized by much of the research reviewed earlier, may actually limit students' development of more sophisticated fraction concepts including fraction division.

The implications for conceptual-first fraction instruction are substantial. If part-whole approaches, which form the conceptual foundation in most elementary fraction curricula and in much of the research supporting conceptual fraction instruction, are themselves limited, then teaching these concepts first may establish inadequate or even counterproductive cognitive foundations for later fraction operations including division. Wilkins and Norton's (2018) research suggests that students who develop measurement-based schemes for understanding unit fractions, proper fractions, and generalized fractions develop more robust and transferable fraction understanding than students whose instruction emphasizes part-whole relationships as the primary conceptual entry point.

This perspective challenges the assumption in the Conceptual vs. Procedural Knowledge Framework that any form of conceptual understanding of fractions provides a superior foundation to procedural knowledge of fraction algorithms. Instead, Wilkins and Norton (2018) suggest that the specific type of conceptual understanding of fractions emphasized in instruction matters significantly. Teaching part-whole fraction concepts before teaching fraction procedures may simply replace one form of limited understanding with another, rather than establishing the strong cognitive foundations for fraction division and other advanced fraction operations that conceptual-first advocates claim. This finding does not argue against conceptual instruction, it argues for more carefully chosen conceptual instruction, one that prioritizes the measurement-based understandings that best prepare students for the full range of fraction operations they will encounter.

Synthesis of Counterpoints: Implications for Conceptual-First Fraction Instruction

Collectively, these four studies do not dismantle the argument for conceptual-first fraction instruction, they sharpen and refine it into a more defensible and nuanced position. The evidence from these counterpoints converges on a conclusion that is actually more powerful than a simple conceptual-first prescription; procedural instruction alone is demonstrably insufficient for meaningful fraction learning, and beginning with conceptual understanding while remaining actively responsive to students' developing knowledge provides the strongest foundation available for fraction division and the mathematical reasoning that follows.

The counterpoints establish four important qualifications. First, conceptual fraction instruction, even when implemented well, may not fully prevent the persistent cognitive biases that interfere with fraction understanding, particularly the whole number bias that most directly undermines reasoning about fraction division (Braithwaite & Siegler, 2018). Second, the theoretical mechanisms underlying whole number bias in fraction learning remain contested, meaning that instructional prescriptions intended to address this bias must be held with appropriate humility and ongoing attention to how individual students are actually responding (Ni & Zhou, 2005). Third, the relationship between conceptual and procedural fraction knowledge is more individualized than a uniform sequential model suggests, with students following varied developmental pathways that do not all begin from the same starting point (Lenz et al., 2020). Fourth, the specific type of conceptual understanding emphasized matters critically, with measurement-based approaches offering stronger foundations for fraction division than traditional part-whole instruction alone (Wilkins & Norton, 2018).

What these qualifications point toward, taken together, is not an abandonment of conceptual-first instruction but a more sophisticated version of it. Procedural instruction alone leaves students without any framework for reasoning about why fraction division behaves as it does, without any tools for recognizing and resolving cognitive conflicts with whole number reasoning, and without the relational understanding necessary to transfer fraction knowledge to ratios, proportional reasoning, and algebra. Beginning with conceptual understanding, particularly measurement-based conceptual understanding of what fraction division means, provides the essential groundwork that makes subsequent procedural learning meaningful rather than arbitrary. Once that conceptual foundation is established, conceptual and procedural knowledge can and should develop in tandem, with each reinforcing the other through the iterative relationship that Rittle-Johnson and colleagues (2001) documented.

The critical variable that these counterpoints add to this picture is the teacher. Because students follow varied developmental pathways in fraction learning, because whole number bias persists in ways that resist uniform instructional solutions, and because the relationship between conceptual and procedural knowledge is more individualized than any single prescribed sequence can fully address, teachers must do more than implement a conceptual-first sequence and move on. They must actively listen to their students' reasoning, assess where each student is in their development of both conceptual and procedural fraction knowledge, and make ongoing instructional decisions that are responsive to what they observe. A student who can draw and explain what $\frac{3}{4} \div \frac{1}{2}$ means but struggles to execute the algorithm efficiently needs different instructional support than a student who can execute the algorithm but cannot explain or represent what the quotient represents. A student who continues to reason from whole number logic when comparing fraction magnitudes needs targeted experiences that create cognitive conflict with those intuitions, not simply more procedural practice. Effective fraction division instruction, then, is not a fixed sequence but a responsive practice, one that begins with conceptual grounding, supports the development of both knowledge types in tandem, and is continuously guided by careful attention to each student's evolving understanding.

Synthesis and Conceptual Model

Across the literature, a coherent pattern emerges specifically in the context of fraction learning and fraction division instruction: conceptual understanding provides the foundation for meaningful mathematical learning (DeWolf et al., 2015; Geller et al., 2017; Lenz et al., 2024), which in turn predicts sustained achievement across mathematical domains including ratios, proportional reasoning, and algebra (Bailey et al., 2012) and supports the development of comprehensive fraction proficiency (Tsai & Li, 2017). The three theoretical frameworks that organize this review; the Conceptual vs Procedural Knowledge Framework, the Mathematics Achievement Prediction Framework, and the Proficiency Development Framework intersect to form an integrated model of mathematical growth that is grounded in and most consequentially demonstrated through the specific case of fraction division instruction. Conceptual instruction in fraction division lays the groundwork for comprehension of what the operation means and why it behaves as it does (Barkin, 2019; Kazemi & Stipek, 2001), comprehension that in turn cultivates the ability to transfer fraction knowledge to more advanced mathematical topics (DeWolf et al., 2015), which is further reinforced through instructional practices that promote ongoing proficiency across fraction concepts (Tsai & Li, 2017).

The research synthesized in this review reveals that the relationship between conceptual and procedural knowledge in fraction learning is complex and iterative (Rittle-Johnson et al., 2001), yet the evidence consistently supports beginning fraction instruction with conceptual foundations rather than procedural algorithms (Byrnes & Wasik, 1991; Siegler & Lortie-Forgues, 2015). Gabriel and colleagues (2013), conducting a componential analysis of the specific conceptual barriers students face in fraction learning, explain why conceptual-first instruction is necessary in fraction contexts; students face multiple interconnected conceptual barriers in fraction learning that cannot be overcome through procedural shortcuts alone, and fraction division is among the most demanding of these because it requires students to simultaneously coordinate their understanding of what fractions represent, what division means, and why inverting and multiplying is mathematically valid (Reeder & Utley, 2017). When teachers address these conceptual challenges in fraction division systematically; moving from concrete representations such as physical partitioning of quantities, through pictorial models such as area diagrams and number lines, to abstract symbolic manipulation of the invert-and-multiply algorithm (Rešić et al., 2016) and when they use varied conceptual strategies that help students compare, explain, and justify their fraction reasoning (Wiest & Amankonag, 2019) while recognizing individual differences in how students develop conceptual and procedural fraction knowledge (Hallett et al., 2010), they create the conditions for both immediate understanding of fraction division and long-term mathematical success.

This synthesis underscores that conceptual-first teaching of fraction division is not merely a pedagogical preference but a developmental necessity grounded in decades of converging evidence across mathematics education research (Geary et al., 2008). Procedural instruction alone; teaching students to invert and multiply without first building conceptual understanding of what fraction division means, leaves students without the cognitive tools to recognize errors, interpret their answers, connect fraction division to the ratios and proportional reasoning that follow, or overcome the whole number bias that research shows persists even with extensive mathematical training (Braithwaite & Siegler, 2018). When teachers prioritize conceptual understanding of fraction division first, they equip students with the cognitive structures needed to connect, apply, and extend their fraction knowledge over time (Geller et al., 2017; Stohlmann et al., 2015). Once that conceptual foundation is established, conceptual and procedural knowledge develop in tandem, each reinforcing the other through the iterative relationship the research documents, but that productive cycle depends on conceptual grounding initiating it. This approach also requires that teachers themselves possess deep conceptual understanding of fraction operations, including fraction division (Copur-Gencturk, 2021; Reeder & Utley, 2017), and that they actively listen to and assess their students' developing understanding in order to make instructional decisions that are responsive to individual learning pathways rather than assuming a uniform developmental sequence. Effective fraction division instruction is therefore not a fixed prescription but a responsive and informed practice, one that begins with conceptual understanding, supports the development of both knowledge types in tandem, and is continuously guided by teachers' knowledge of both the mathematics and their students. This integrated body of evidence establishes the theoretical and empirical foundation for examining how conceptual-first instruction in fraction division can promote long-term mathematical success in upper elementary classrooms and beyond.

Chapter 3

Discussion and Recap of Chapter 2

The literature synthesized in chapter 2 reveals both a compelling case for conceptual-first fraction instruction and important complexities that temper overly confident prescriptions. Across three interconnected theoretical frameworks, a consistent pattern emerges; students who develop deep conceptual understanding of fractions and fraction division specifically before engaging with procedural algorithms demonstrate greater problem-solving flexibility, stronger knowledge transfer, and more durable long-term mathematical achievement than students whose instruction prioritizes procedural fluency first.

The Conceptual vs. Procedural Knowledge Framework establishes that instructional sequencing matters cognitively. When students encounter algorithmic procedures such as the invert-and-multiply algorithm for fraction division before developing relational understanding, they construct fragmented mental models that limit their ability to reason across mathematical contexts. The iterative relationship documented by Rittle-Johnson and colleagues confirms that conceptual and procedural knowledge reinforce one another, but the initial conceptual grounding provides the strongest scaffold for this reciprocal development (Rittle-Johnson et al., 2001; Rittle-Johnson et al., 2015). The Mathematics Achievement Prediction Framework extends this argument longitudinally, demonstrating that fraction competence in upper elementary school functions as a critical bridge to algebraic reasoning, proportional thinking, and advanced mathematics. Students who understand why fraction operations work, including why dividing by a fraction produces a larger quotient, not merely how to execute them, are better positioned to navigate the increasingly abstract demands of middle and high school mathematics. The Proficiency Development Framework completes the picture by situating these cognitive and predictive dimensions within the realities of classroom instruction, highlighting that sustainable fraction learning and fraction division learning specifically depends on intentional pedagogical design, differentiated support, and teachers who themselves possess deep conceptual knowledge.

However, the counterpoints examined introduce necessary nuance. Braithwaite and Siegler's documentation of whole number bias persisting well into adolescence and most directly interfering with students' reasoning about fraction division, where the counterintuitive results of dividing by a fraction less than one most directly contradict whole number logic, challenges the assumption that conceptual instruction alone can override deeply rooted cognitive interference. Lenz and colleagues' findings regarding individual variation in developmental pathways complicate any uniform prescription for sequencing. Ni and Zhou's theoretical analysis reveals that the cognitive mechanisms underlying fractions difficulties remain incompletely understood, and Wilkins and Norton's critique raises the possibility that part-whole instruction, the cornerstone of most elementary fraction curricula, may itself establish limited foundations compared to measurement-based alternatives.

Taken together, these perspectives suggest that the question is not simply whether to teach conceptually before procedurally, but how to explicitly pay attention to whole number bias, individual developmental differences, incorporate multiple fraction interpretations including measurement, and ongoing formative assessment that informs instructional decisions. The research does not support abandoning conceptual-first instruction; it supports making it more sophisticated.

A commonality running throughout chapter 2 that deserves particular emphasis is the role of teacher knowledge. Multiple studies converge on the finding that effective conceptual instruction is inseparable from teachers' own conceptual understanding. When teachers lack the mathematical depth to explain why procedures work, respond meaningfully to students' reasoning, or recognize and address misconceptions as they emerge, even well-designed curricula are likely to default toward procedural emphasis. This finding positions teacher preparation not as a peripheral concern but as a central condition for the instructional improvements this literature advocates.

Ultimately, the evidence reviewed in chapter 2 affirms that fraction instruction in upper elementary classrooms occupies a pivotal role in students' mathematical trajectories. The persistent difficulties students experience with fractions are not inevitable. They reflect, in significant part, instructional choices about what to emphasize, in what order, and with what depth. The frameworks synthesized here provide both a rationale and a roadmap for making those choices more deliberately; and the counterpoints ensure that the roadmap is drawn with appropriate humility about what remains unknown.

Recommendations for Classroom Teachers

1. Prioritize Conceptual Understanding While Recognizing Its Complexity

Teachers should begin fraction instruction with conceptual foundations, but must recognize that conceptual understanding alone does not guarantee the prevention of persistent cognitive challenges. Research demonstrates that conceptual-first instruction supports procedural learning (Byrnes & Wasik, 1991), enhances problem-solving ability (Siegler & Lortie-Forgues, 2015), and predicts long-term mathematical achievement (Bailey et al., 2012). However, whole number bias persists even among students who have received conceptual instruction (Braithwaite & Siegler, 2018), indicating that teachers must explicitly address cognitive interference from whole number knowledge structures throughout fraction division instruction in particular, where the counterintuitive results of dividing by a fraction less than one most directly contradict students' whole number reasoning.

More specifically, fifth and sixth grade teachers should begin each fraction division unit with activities that build conceptual understanding through multiple representations before the invert-and-multiply algorithm is ever introduced. A concrete starting point is to present students with a real-world partitive division context: if you have $\frac{3}{4}$ of a cup of rice and each serving is $\frac{1}{2}$ a cup, how many servings do you have? Ask students to solve this problem first by drawing it, using a number line or an area model to physically show how many groups of $\frac{1}{2}$ fit into $\frac{3}{4}$ before any symbolic notation is introduced. This experience gives students a mental image of what fraction division means that they can return to when the algorithm is introduced later (Rešić et al., 2016). After students have represented and discussed several such problems visually, introduce the symbolic expression $\frac{3}{4} \div \frac{1}{2}$ and ask students to connect it to their drawing. Only after students can fluently move between the visual representation and the symbolic expression should the invert-and-multiply procedure be introduced, and when it is, it should be presented as an efficient shortcut for something students already understand rather than as a rule to memorize.

Design experiences throughout the unit that create cognitive conflict with whole number assumptions about division. A particularly effective strategy for fifth and sixth grade students is to ask them to predict the answer to a fraction division problem before solving it, and then compare their prediction to the actual result. For example, ask students to predict whether $\frac{3}{4} \div \frac{1}{2}$ will be greater than or less than $\frac{3}{4}$ before they solve it. Most students will predict a smaller result based on whole number reasoning, and the cognitive conflict that arises when they discover the quotient is larger than the dividend creates a powerful teachable moment. Follow this with explicit discussion of why dividing by a fraction less than one produces a larger result, using the visual model to justify the answer. Revisit this type of task periodically throughout the unit, since research shows that a single exposure is insufficient to overcome deeply rooted whole number bias (Braithwaite & Siegler, 2018).

Implementing the iterative model of knowledge development means structuring fraction division instruction as a deliberate cycle rather than a linear progression from concepts to procedures. In practice this looks like the following sequence. Begin a lesson or unit with a conceptual exploration, such as the real-world partitive problem described above, where students represent and discuss what fraction division means using visual models. After students have built conceptual understanding through this exploration, introduce the procedural algorithm and have students practice it while keeping their visual models visible and referring back to them. Then return to a conceptual task, such as asking students to create their own real-world story problem that matches a given fraction division expression. This will deepen and consolidate the conceptual understanding that was established before procedural practice. Follow this with additional procedural practice, and then return again to a conceptual task such as asking students to use a number line to explain why their answer makes sense. Each phase in this cycle builds upon and reinforces the other; the conceptual exploration gives meaning to the procedure, the procedural practice builds fluency that frees up cognitive resources for deeper conceptual reasoning, and the return to conceptual tasks ensures that procedural fluency does not become disconnected from meaning. This cyclical approach prevents the common pattern where students learn the invert-and-multiply procedure in isolation, can execute it correctly on a worksheet, but cannot explain what their answer means or apply their knowledge when fraction division appears in a ratio or proportional reasoning context in later grades (Rittle-Johnson et al., 2001).

2. Emphasize Measurement Concepts Alongside Part-Whole Relationships

While part-whole understanding has traditionally formed the foundation of elementary fraction instruction, research suggests these concepts may limit rather than support students' development of sophisticated fractional understanding, particularly for fraction division (Wilkins & Norton, 2018). Teachers should incorporate measurement-based approaches that emphasize partitioning and iterating, helping students coordinate the three levels of units inherent in fractions. This measurement framework supports more robust and transferable understanding of fraction division than part-whole approaches alone, because it builds the conceptual foundation students need to understand what it means to find how many groups of one fractional quantity are contained in another.

More specifically, fifth and sixth grade teachers should design activities where students partition continuous quantities into equal parts and then iterate those parts to create quantities greater than one whole before fraction division is introduced symbolically. For example, give students a length of ribbon or a drawn number line representing one whole and ask them to fold or partition it into thirds. Then ask them how many two-thirds fit into the whole ribbon, requiring them to physically iterate the two-thirds unit along the length. This experience builds the conceptual foundation for understanding fraction division as a measurement question, how many of this unit fit into that quantity, which is precisely what $\frac{3}{4} \div \frac{1}{2}$ is asking. Once students can answer these questions physically and with drawings, connect the activity explicitly to the symbolic expression and eventually to the invert-and-multiply algorithm, showing how the algorithm efficiently answers the measurement question students have already been reasoning about concretely.

Introduce fraction division through multiple interpretations rather than part-whole relationships alone. The quotient interpretation, understanding a fraction as the result of division, is especially important for fraction division in fifth and sixth grade. This interpretation helps students understand why dividing $\frac{3}{4}$ by $\frac{1}{2}$ can be thought of as asking how many halves are in three-quarters. The measurement interpretation similarly supports this reasoning by framing division as finding how many equal-sized groups of a given fractional amount are contained in a larger quantity. Present students with real-world contexts that naturally invoke these interpretations: how many half-hour segments fit into a three-quarter hour period, or how many half-cup scoops fill a container that holds three-quarters of a cup. Students who engage with fraction division through these multiple interpretive lenses develop more flexible understanding that supports later work with ratios, proportions, and algebra than students whose fraction understanding is limited to part-whole relationships (Pitkethly & Hunting, 1996).

3. Differentiate Instruction Based on Individual Developmental Pathways

Research conducted specifically with students learning fraction concepts reveals significant individual differences in how students develop conceptual and procedural knowledge of fractions (Hallett et al., 2010; Lenz et al., 2020). Rather than assuming all fifth and sixth grade students benefit equally from identical instructional sequences for fraction division, teachers should recognize that some students demonstrate stronger initial conceptual understanding while others show procedural facility first. The evidence suggests that students who begin with stronger conceptual knowledge of fraction concepts show greater long-term gains in both conceptual and procedural domains, but individual pathways vary considerably and teachers must actively assess and respond to where each student is in their development rather than assuming a uniform progression.

More specifically, before beginning a fraction division unit, assess students' existing conceptual and procedural knowledge to identify starting points and knowledge profiles. A useful pre-assessment for fraction division asks students to do three things with the same problem such as $\frac{3}{4} \div \frac{1}{2}$: solve it using any method they choose, draw a picture or diagram that represents what the problem means, and write a real-world story that the problem could represent. This three-part task quickly reveals whether a student has conceptual understanding of what fraction division means, procedural knowledge of the algorithm, both, or neither, and gives the teacher a clear picture of where to begin instruction for each student.

Use formative assessment throughout fraction division instruction to monitor how students are developing both types of knowledge, adjusting instructional emphasis accordingly. During class discussions, ask students to both solve fraction division problems and explain what their answer means in a real-world context. A student who can execute the algorithm but cannot interpret the quotient is signaling a need for more conceptual work, while a student who can draw and explain the problem but makes consistent procedural errors needs a targeted practice with the algorithm while maintaining connections to the visual model (Hallett et al., 2010; Lenz et al., 2020). For students who demonstrate strong procedural facility with the invert-and-multiply algorithm, but weak conceptual understanding of what fraction division means, provide explicit opportunities to connect their procedural knowledge to visual representations by requiring them to draw a number line or area model that matches their symbolic answer before the answer is accepted as complete. For students who demonstrate strong conceptual understanding of fraction division through accurate drawings and explanations but struggle with procedural execution of the algorithm, offer structured practice opportunities that keep the visual model present alongside the symbolic procedure rather than moving to isolated algorithmic drills. Recognize that the goal is not uniform procedural execution at the expense of conceptual understanding but rather developing both forms of fraction knowledge in ways that are responsive to each individual student's current understanding and learning trajectory.

4. Foster Mathematical Discourse That Makes Reasoning Visible

Classrooms that emphasize mathematical discussion, reasoning, and student exploration of fraction concepts promote stronger conceptual gains than those focused primarily on procedural execution (Kazemi & Stipek, 2001). When students are asked to provide conceptual explanations for their fraction division solutions specifically, they demonstrate more durable and transferable understanding than students who simply calculate answers (Geller et al., 2017). Creating classroom environments in fifth and sixth grade where students justify their fraction division reasoning, compare solution strategies, and connect visual representations to symbolic expressions deepens understanding and supports the development of mathematical discourse practices essential for success in the ratios, proportional reasoning, and algebra that follow.

More specifically, regularly ask students to explain their fraction division reasoning using structured sentence stems that require them to connect their visual model to their symbolic answer. For example: "I know that $\frac{3}{4} \div \frac{1}{2}$ equals $1\frac{1}{2}$ because when I look at my number line, I can see that…" or "My drawing shows that there are $1\frac{1}{2}$ groups of $\frac{1}{2}$ in $\frac{3}{4}$, which means the answer makes sense because…" These sentence stems require students to do more than state an answer, they require students to connect the procedure they executed to the conceptual meaning of the problem, which is precisely the connection that makes fraction division understanding durable and transferable.

Give students regular opportunities to compare multiple solution strategies for fraction division problems, discussing the advantages and limitations of different approaches. For example, present two student work samples side by side; one that solved $\frac{3}{4} \div \frac{1}{2}$ using the invert-and-multiply algorithm and one that solved it by drawing a number line and ask the class to discuss what each approach shows. Ask the class which approach makes it easier, and which approach makes it harder to see. This type of comparison task makes mathematical reasoning visible and helps students understand that the algorithm and the visual model are two representations of the same mathematical relationship rather than two separate procedures. Avoid rushing to closure when students are puzzling through conceptual challenges in fraction division. Allow productive struggle that supports genuine understanding rather than offering the algorithm as a shortcut before students have developed the conceptual foundation that makes the algorithm meaningful. Establish classroom norms that value explanation and justification of fraction reasoning as much as correct answers, creating a mathematical community in fifth and sixth grade where sense-making about what fraction division means is the central goal rather than procedural speed and accuracy.

5. Address Common Misconceptions Explicitly and Systematically

Fifth and sixth grade students commonly overgeneralize whole number reasoning to fraction division context, creating persistent misconceptions that interfere with their ability to understand and apply fraction division meaningfully (Syed Ismail et al., 2024). These misconceptions do not resolve simply through exposure to the correct invert-and-multiply procedure. Students who are taught the algorithm without conceptual grounding frequently continue to hold contradictory intuitions about what fraction division should produce, making errors when problems appear in unfamiliar formats or real-world contexts. Addressing these misconceptions requires explicit instructional attention that creates cognitive conflict with whole number assumptions and supports genuine conceptual reorganization of how students think about division when fractions are involved.

More specifically, anticipate and explicitly address the three misconceptions most directly relevant to fraction division in fifth and sixth grade. The first is the belief that division always makes quantities smaller; a deeply rooted whole number intuition that is directly contradicted by fraction division, where dividing by a fraction less than one always produces a quotient larger than the dividend. Address this misconception head-on before introducing the algorithm by asking students to predict whether $\frac{3}{4} \div \frac{1}{2}$ will be greater than or less than $\frac{3}{4}$, recording their predictions, and then using a number line to demonstrate that the quotient is $1\frac{1}{2}$, which is larger than $\frac{3}{4}$. Make the cognitive conflict explicit: "We know that 6 ÷ 2 = 3, which is smaller than 6. But $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$, which is larger than $\frac{3}{4}$. Why does division behave differently here?" Use the visual model to answer this question conceptually before moving to the symbolic procedure.

The second misconception is that fractions with larger whole number components are always greater in value, which interferes with students' ability to reason about the quantities involved in fraction division. Address this by presenting students with pairs of equivalent fractions such as $\frac{6}{8}$ and $\frac{3}{4}$ and asking them to determine which is greater before solving any division problems, using area models to verify their reasoning. The third misconception is that the invert-and-multiply procedure works by magic rather than by mathematical logic, which leaves students unable to catch their own errors or adapt when problems are presented differently. Address this by explicitly connecting the algorithm to the visual model. Show students that multiplying by the reciprocal of $\frac{1}{2}$ is the same as asking how many halves fit into the dividend, which is exactly what their number line drawing showed. Revisit all three of these misconceptions periodically throughout the fraction division unit and again when ratios and proportional reasoning are introduced, recognizing that single exposures are insufficient to overcome the deeply rooted whole number bias that research shows persists well into adolescence (Braithwaite & Siegler, 2018).

Recommendations for Teacher Preparation Programs

1. Strengthen Prospective Teachers Conceptual Understanding of Fractions

Research conducted specifically with prospective elementary teachers reveals that many struggle to articulate what a fraction is beyond procedural definitions and cannot provide conceptual justifications for fraction operations including division (Reeder & Utley, 2017). When asked to explain why the procedure for dividing fractions works, why inverting the second fraction and multiplying produces the correct quotient, many pre-service teachers resort to restating the steps of the algorithm rather than explaining the underlying mathematical logic. Copur-Gencturk (2021), surveying a national sample of practicing elementary teachers specifically about their conceptual understanding of fraction operations, confirmed that this gap is not limited to novice teachers but persists broadly across the profession. This fundamental gap in mathematical content knowledge severely limits teachers' capacity to implement conceptual-first fraction division instruction, regardless of their pedagogical training, because teachers who cannot themselves explain why the invert-and-multiply algorithm works cannot facilitate the conceptual discussions and visual reasoning that research shows are essential for student understanding.

Teacher preparation programs should design mathematics content courses specifically for elementary education majors that prioritize conceptual development of fraction operations over procedural review. These courses should include extensive work with the visual representations and real-world contexts that are most relevant to fifth and sixth grade fraction division instruction; number lines, area models, and measurement contexts. These courses would ensure that prospective teachers can fluently move among these representations and connect each one to the symbolic algorithm. A particularly powerful preparation activity is to require prospective teachers to solve fraction division problems such as $\frac{3}{4} \div \frac{1}{2}$ in three ways: by drawing a number line that shows how many groups of $\frac{1}{2}$ fit into $\frac{3}{4}$, by constructing a real-world story problem that the expression represents, and by executing and explaining the invert-and-multiply algorithm with reference to the visual model. Requiring prospective teachers to explain why fraction division algorithms work using visual models and conceptual reasoning, not just demonstrate procedural execution, develops the depth of understanding they will need to answer the why questions their fifth and sixth grade students will ask. This exercise will also help novice teachers design instruction that builds conceptual understanding before introducing the algorithm.

2. Develop Pedagogical Content Knowledge Specific to Fraction Division Instruction

Understanding fraction division conceptually is necessary but insufficient for effective teaching of fraction division in fifth and sixth grade. Prospective teachers also need pedagogical content knowledge that encompasses understanding how children construct fractional knowledge through progressive schemes from partitioning activities to abstract reasoning (Steffe, 2001), how teacher facilitation specifically impacts whether students develop conceptual understanding of improper fractions and fraction division or resort to procedural manipulation (Tzur, 1999), recognizing the specific conceptual barriers students face in fraction learning and the common misconceptions that arise when whole number reasoning is overgeneralized to fraction division contexts (Gabriel et al., 2013; Pitkethly & Hunting, 1996), and designing instruction that supports conceptual development of fraction division through discussion, visual representations, and real-world problem contexts (Kazemi & Stipek, 2001; Zhang et al., 2020). This specialized pedagogical content knowledge distinguishes between knowing fraction division mathematics and knowing how to teach fraction division effectively to fifth and sixth grade students (Copur-Gencturk, 2021; Reeder & Utley, 2017).

Teacher preparation programs should include coursework and field experiences that develop this pedagogical content knowledge specifically in the context of fraction division. Prospective teachers should practice facilitating classroom discussions in which students explain and justify their fraction division reasoning using visual models, developing their ability to ask probing questions that elicit student thinking rather than simply confirming correct answers. They should practice analyzing student work samples from fraction division tasks to identify whether errors reflect conceptual misunderstanding of what division by a fraction means or procedural errors in executing the algorithm, since these two types of errors require fundamentally different instructional responses. They should also practice designing lesson sequences for fraction division that follow the concrete-pictorial-abstract progression identified by Rešić and colleagues (2016) as essential for effective fraction instruction, ensuring they can move students from physical partitioning of quantities through visual models to the symbolic algorithm in a way that builds and maintains conceptual understanding at each stage.

3. Challenge and Reshape Prospective Teachers’ Beliefs About Mathematics Teaching

Many prospective teachers enter preparation programs with beliefs about mathematics teaching shaped by their own procedurally focused fraction education. Having learned fraction division as the invert-and-multiply rule without conceptual grounding, they may view mathematics as a set of procedures to be memorized and transmitted rather than a sense-making discipline to be explored and constructed, and they may assume that effective teaching of fraction division means clearly demonstrating the algorithm rather than facilitating the conceptual understanding that makes the algorithm meaningful. Research studying pre-service teachers' beliefs about mathematics instruction broadly found that those who shifted their views toward mathematics as conceptual sense-making were significantly more likely to engage students in meaningful learning experiences (Stohlmann et al., 2015). For fraction division specifically, this belief shift is essential, a teacher who views the invert-and-multiply procedure as the destination of fraction division instruction will structure lessons very differently from a teacher who views conceptual understanding of fraction division means as the destination, with the algorithm as one efficient tool for expressing that understanding symbolically.

Teacher preparation programs should explicitly surface and examine prospective teachers' beliefs about fraction division instruction by asking them to reflect on how they themselves learned fraction division, what they remember understanding versus what they remember memorizing, and what they wish they had understood conceptually. These reflections create productive entry points for introducing research on why conceptual-first fraction instruction produces better long-term outcomes than procedural-first approaches (Byrnes & Wasik, 1991; Rittle-Johnson et al., 2001). Programs should provide prospective teachers with opportunities to experience fraction division conceptually themselves, working through the real-world measurement contexts, visual models, and partitioning activities that they will eventually use with students. These opportunities will shift prospective teachers' beliefs to be grounded in genuine mathematical experience rather than abstract principle. When prospective teachers experience the conceptual foundation of fraction division firsthand and recognize what they were missing in their own procedural education, they are far better positioned to prioritize that foundation for their own students.

Recommendations for School Administrators

School administrators play a critical role in creating the structural conditions that make conceptual-first fraction division instruction possible in fifth and sixth grade classrooms. When teachers feel pressured to cover extensive standards quickly, they resort to procedural shortcuts such as teaching the invert-and-multiply algorithm without conceptual grounding, which may produce short-term performance gains at the expense of the long-term understanding that research consistently shows predicts success in algebra and advanced mathematics (DeWolf et al., 2015). Administrators must therefore make deliberate structural decisions that protect the time and professional support teachers need to implement conceptual fraction division instruction effectively.

Administrators should allocate extended instructional time for fraction units in fourth through sixth grades, recognizing that fraction competence, and fraction division specifically, represents one of the most important predictors of later mathematical success (Bailey et al., 2012). Building the conceptual foundation for fraction division moving students from physical partitioning through visual models to the symbolic algorithm in a way that maintains conceptual understanding at each stage cannot be rushed without undermining the very outcomes that make the investment worthwhile (Rešić et al., 2016). This requires restructuring curriculum pacing guides to allow depth over breadth in fraction units, and having explicit conversations with teachers about why the time allocated to conceptual fraction division instruction is justified by the long-term mathematical benefits rather than evaluating instructional effectiveness solely on the basis of how quickly teachers move through standards.

Administrators should also prioritize and fund professional development specifically focused on teachers' conceptual understanding of fraction division and their capacity to implement conceptual-first instruction. Given that research documents significant gaps in practicing elementary teachers' conceptual understanding of fraction operations including division (Copur-Gencturk, 2021), professional development that simply introduces new curricular materials without developing teachers' own conceptual knowledge is unlikely to produce meaningful change in classroom instruction. Effective professional development for fraction division instruction should engage teachers in the same conceptual experiences they will provide to students; working with visual models, constructing real-world contexts, and explaining why the invert-and-multiply algorithm works while developing their ability to recognize and respond to the specific misconceptions and developmental progressions that fifth and sixth grade students exhibit when learning fraction division.

Finally, administrators should evaluate instructional effectiveness in fraction division based on evidence of student conceptual understanding and long-term retention rather than solely on immediate procedural test performance. This means supporting teachers in using formative assessment practices that reveal students' conceptual understanding of fraction division, such as asking students to draw and explain fraction division problems rather than only solve them algorithmically. It also means valuing the evidence these assessments provide about the quality of fraction instruction even when it is not captured by standardized procedural measures. When administrators signal through their evaluation practices that conceptual understanding of fraction division matters as much as procedural accuracy, they create the professional culture in which teachers feel supported to prioritize the conceptual-first instruction that research shows produces the strongest long-term mathematical outcomes for students.

Synthesis of Recommendations

The recommendations presented reflect both the substantial evidence supporting conceptual-first division instruction and the important qualifications and complexities that this literature review has identified. Improving fraction division instruction in fifth and sixth grade requires coordinated action across multiple levels of the educational system, because no single change; whether in classroom practice, teacher preparation, or administrative structure is sufficient on its own to produce the sustained mathematical outcomes that research shows are possible when fraction division is taught conceptually.

Classroom teachers must implement conceptual-first approaches to fraction division by building students' understanding of what division by a fraction means before introducing the invert-and-multiply algorithm, using visual models and real-world measurement contexts to make fraction division meaningful rather than arbitrary. At the same time, teachers must recognize that conceptual instruction alone does not automatically resolve the persistent cognitive challenges students face. Whole number bias in particular requires explicit and repeated instructional attention throughout fraction division instruction, not a single lesson (Braithwaite & Siegler, 2018). Teachers must differentiate their instruction based on ongoing formative assessment of where individual students are in their development of both conceptual and procedural fraction knowledge, actively listening to student reasoning and responding to what they observe rather than assuming all students follow an identical developmental pathway. And teachers must incorporate measurement-based conceptual approaches alongside part-whole relationships, ensuring that the conceptual foundation they build for fraction division is robust enough to support the ratios, proportional reasoning, and algebra that depend on it in later grades.

Administrators must provide the structural conditions that make this quality of fraction division instruction possible; extended instructional time in fraction units that allows conceptual understanding to be built before algorithms are introduced, professional development that develops teachers' own conceptual understanding of fraction division alongside their pedagogical capacity to facilitate it, and evaluation practices that value evidence of student conceptual understanding rather than procedural performance alone. Teacher preparation programs must develop both the mathematical content knowledge prospective teachers need to explain why fraction division works conceptually and the pedagogical content knowledge they need to facilitate that understanding in fifth and sixth grade classrooms, while also challenging the procedurally focused beliefs about mathematics teaching that many prospective teachers bring from their own educational experiences.

Most fundamentally, all stakeholders must recognize that teaching fraction division conceptually before procedurally is not merely a matter of reordering instructional sequences but of transforming the mathematical experiences provided to students. Procedural instruction alone; teaching students to invert-and-multiply without first building conceptual understanding of what fraction division means, is demonstrably insufficient for producing the deep, flexible mathematical understanding that predicts long-term success (Lenz et al., 2024; Barkin, 2019). Beginning with conceptual understanding of fraction division, supporting the development of both conceptual and procedural knowledge in tandem through carefully designed iterative instruction, and remaining continuously responsive to individual students' developing understanding together represent the most evidence-supported path toward meaningful fraction learning. This transformation requires challenging long-standing assumptions about mathematics teaching, investing seriously in teacher knowledge development at both the preparation and professional development levels, and committing to instructional approaches that prioritize long-term conceptual understanding over short-term procedural performance.

The persistent challenges students face with fraction division despite decades of instruction indicate that incremental modifications to existing procedurally focused approaches are insufficient. The evidence synthesized in the review demonstrates that conceptual understanding of fraction division provides the essential foundation for meaningful fraction learning, predicts long-term mathematical achievement across algebra and advanced mathematics (Bailey et al., 2012), and supports knowledge transfer to the ratios and proportional reasoning that depend on fraction understanding. However, this evidence also reveals that conceptual instruction is more complex than simple sequential models suggest, requiring explicit attention to persistent cognitive challenges such as whole number bias, recognition of individual developmental differences in how students build fraction knowledge, careful selection of which conceptual foundations; measurement-based rather than part-whole alone, best prepare students for fraction division, and teachers who actively listen to and assess their students throughout instruction rather than following a fixed prescriptive sequence.

By implementing these recommendations with attention to both the compelling evidence for conceptual-first fraction division instruction and the important nuances that research has revealed, educators at all levels can better support fifth and sixth grade students in developing not just procedural proficiency with fraction division but the deep, flexible understanding of fractions that prepares them for sustained mathematical success in algebra and beyond.

Personal Reflection: Twenty Years of Teaching

As I reflect on two decades of elementary teaching, fourteen of those years spent in fifth grade, my journey with fraction instruction mirrors the very tensions this literature review has explored. When I first began teaching fifth grade, I approached fractions the way I had been taught; through procedural shortcuts, mnemonics, and "tricks" designed to help students arrive at correct answers quickly. "Leave it, change it, flip it" for dividing fractions; a mnemonic that gave my students a procedure for fraction division without any understanding of what dividing by a fraction actually means or why the algorithm produces the result it does, and others became staples of my instruction. These methods produced short-term results on assessments, and I believed I was serving my students well by giving them tools to succeed.

However, seven years ago, I began a fundamental shift in my instructional approach. As I pursued deeper mathematical content knowledge on my own and then through my master's program, I started recognizing the conceptual foundations I had never developed; either in my own education or in my teaching practice. I began emphasizing conceptual understanding before procedural execution, starting with fraction division as the most glaring example of a concept I had taught procedurally for years without conceptual grounding, using multiple representations, and encouraging mathematical discourse that made students' reasoning visible. The transformation in my classroom has been remarkable. Over these seven years, I have observed not only increases in state assessment scores but, more importantly, a qualitatively different kind of understanding in my students. They can explain why procedures work, transfer their knowledge to novel problems, and demonstrate the kind of flexible mathematical thinking that research suggests comes from strong conceptual foundations.

This personal experience initially led me to this literature review with a sense of conviction about conceptual-first instruction. The research by Bailey et al. (2012), Siegler and Lortie-Forgues (2015), and others seemed to validate what I had witnessed in my classroom: that conceptual understanding provides the foundation for meaningful mathematical learning and predicts long-term success. However, as this review progressed, particularly through the examination of counterpoints and alternative perspectives, I found myself confronting a more complex reality than I had anticipated.

The research reveals multiple, sometimes competing explanations for why students struggle with fractions. Braithwaite and Siegler's (2018) documentation of whole number bias persisting well into adolescence and even adulthood challenges my initial optimism about conceptual instruction as a comprehensive solution. Their findings suggest that even with strong conceptual foundations, students may continue to experience cognitive interference from whole number knowledge structures. Similarly, Ni and Zhou's (2005) analysis of competing theoretical accounts reveals fundamental disagreements about the cognitive mechanisms underlying fraction learning, making it difficult to prescribe instructional approaches with absolute confidence. Perhaps most troubling is Wilkins and Norton's (2018) critique suggesting that the part-whole conceptual understanding I have emphasized, and that forms the foundation of most elementary curricula, may itself be limited. Their argument for measurement-based approaches raises the possibility that even conceptual-first instruction, if focused on the wrong concepts, may not establish the robust foundations students need for later mathematical success. This realization has been both humbling and enlightening.

These counterpoints have not diminished my commitment to conceptual instruction, but they have deepened my understanding of its complexities. I now recognize that the improvements I have observed in my classroom may result from multiple factors: the specific types of conceptual understanding I emphasize, the way I sequence and integrate procedural practice, my ability to recognize and address individual differences in students' developmental pathways, and perhaps most importantly, the explicit attention I now give to challenging students' whole number assumptions through carefully designed cognitive conflicts.

This research journey has also expanded my concerns beyond fractions to the broader landscape of elementary mathematics instruction. After twenty years in the classroom, I have increasingly questioned whether we teach too many skills too quickly, allowing too many shortcuts and tricks along the way. The pressure to cover extensive standards in limited time creates systemic incentives for procedural efficiency over conceptual depth. I wonder whether slowing down, ensuring that teachers can teach their content deeply, and that students can construct robust conceptual foundations before moving forward would better serve students as they progress through elementary school.

This wondering connects directly to the critical role of teacher preparation identified in this literature review. Copur-Gencturk's (2021) finding that many elementary teachers lack the deep conceptual understanding necessary to teach fractions meaningfully resonates with my own experience. For my first seven years teaching fifth grade, I was one of those teachers. I could execute fraction procedures but could not explain why they worked, which fundamentally limited the instruction I could provide. My transformation as a teacher required not just learning new pedagogical techniques but developing my own conceptual mathematical knowledge, the same knowledge I now work to build in my students.

The implications extend to how we prepare and support teachers. If effective fraction instruction depends on teachers possessing deep conceptual understanding, knowledge of children's developmental progressions, and pedagogical skills to facilitate conceptual construction, then teacher preparation and professional development must prioritize these dimensions. Curriculum alone cannot solve the fraction learning problem if teachers lack the knowledge and confidence to implement conceptual instruction effectively.

As I conclude this literature review, I am left with a more nuanced understanding than I began with. There is no single, simple solution to the challenge of teaching fractions. The research does not provide unequivocal prescriptions but rather reveals the complexity of the cognitive and instructional issues involved. However, this complexity does not lead to paralysis. Despite the counterpoints and uncertainties, the preponderance of evidence still supports beginning fraction instruction with strong conceptual foundations, while recognizing that:

- Conceptual instruction must explicitly address whole number bias through experiences that create cognitive conflict and resolution, not merely present alternative representations.
- Individual students may develop conceptual and procedural knowledge through different pathways, requiring teachers to recognize and accommodate these differences rather than rigidly adhering to a single instructional sequence.
- The specific concepts emphasized matter critically; measurement-based understandings may provide more robust foundations than part-whole relationships alone.
- Conceptual-first instruction is necessary but not sufficient; it must be combined with strategic procedural practice, attention to knowledge transfer, and ongoing assessment of students' developing understanding.

- Teachers must possess the deep conceptual knowledge and pedagogical skills necessary to implement this instruction effectively, highlighting the importance of comprehensive teacher preparation.

My twenty years of teaching experience, combined with this research review, has convinced me that improving fraction instruction and elementary mathematics education more broadly requires systemic changes at multiple levels. We need a curriculum that prioritizes conceptual depth over procedural coverage. We need teacher preparation programs that develop educator's mathematical content knowledge alongside their pedagogical skills. We need professional development that helps practicing teachers deepen their own conceptual understanding and learn to recognize students' mathematical thinking. We also need educational policies that allow teachers the time and support necessary to teach mathematics deeply rather than merely covering the standards superficially.

Most importantly, we need to recognize that teaching fractions conceptually before procedurally is not just about changing instructional sequences but about fundamentally transforming the mathematical experiences we provide students. It requires creating classroom environments where students actively construct understanding through exploration, discussion, and reflection rather than passively receiving procedures to memorize. This transformation challenges long-standing assumptions about mathematics teaching and learning, but the potential benefits; students who understand mathematics deeply, reason flexibly, and achieve sustained success make the effort worthwhile.

As I return to my fifth-grade classroom, I do so with renewed commitment to conceptual instruction, tempered by deeper awareness of its complexities and challenges. The research has not provided simple answers, but it has equipped me with more sophisticated questions to guide my practice: How can I help students construct measurement-based understandings alongside part-whole concepts? How can I help my students understand what fraction division actually means before I show them how to execute the algorithm? How can I design instruction that explicitly challenges whole number bias? How can I recognize and support the varied developmental pathways my students follow? How can I continue developing my own conceptual understanding so I can better facilitate my students' learning? These questions will shape my ongoing work as I continue striving to provide my students with the mathematical foundations they deserve.

Limitations

As the author of this extended literature review, I recognize several important limitations in both my methodology and the research base I have drawn upon, and I believe it is essential to name them honestly rather than allow the confidence of my recommendations to obscure the boundaries of what this review can reliably claim.

The most fundamental limitation is the nature of the methodology itself. Because I am synthesizing and interpreting existing research rather than generating new empirical data, my conclusions are shaped by the sources I selected and the interpretive choices I made throughout the review process. Although I made deliberate efforts to include both supporting evidence and counterpoints, my own positionality as a classroom teacher who has experienced meaningful results from conceptual-first instruction likely influenced how I weighted competing findings. A researcher approaching this literature from a different vantage point might have emphasized different studies or drawn different conclusions from the same body of work. I also acknowledge that publication bias in academic literature tends to favor studies with positive or significant findings, which means the research available to me may already skew toward validating conceptual instruction more strongly than the complete empirical picture would support.

I also recognize significant variation across the studies I reviewed in terms of how conceptual and procedural knowledge are defined and measured. This inconsistency made direct comparison genuinely difficult. This was particularly evident in studies examining fraction division specifically, where some researchers assessed conceptual understanding through visual representation tasks while others used explanation tasks or transfer problem, making it difficult to draw uniform conclusions about what conceptual understanding of fraction division looks like and how it develops across different instructional contexts. I want to be transparent that some of the synthesis I have offered required interpretive bridging across studies that did not use identical frameworks or assessments. Where I have drawn connections between findings, those connections reflect my best scholarly judgment, but they are not always supported by direct replication or standardized measurement across studies.

Additionally, much of the research I reviewed was conducted in specific national contexts, and I cannot confidently assert that all findings translate equally to every classroom environment. While I teach in the United States and have oriented my recommendations accordingly, I want to acknowledge that differences in curriculum design, instructional culture, teacher preparation systems, and student demographics may affect how these findings apply in practice. Similarly, this review is scoped specifically to upper elementary instruction, and I have been careful not to extend its conclusions beyond that range, though I recognize that fraction learning begins earlier and continues later than the grades I have focused on here.

The counterpoints I examined in Chapter 2 also exposed the boundaries of my own initial assumptions in ways I found genuinely humbling. The theoretical uncertainties identified by Ni and Zhou (2005), the persistence of whole number bias documented by Braithwaite and Siegler (2018), and the critique of part-whole instruction offered by Wilkins and Norton (2018) all challenged frameworks I had entered this review prepared to defend with more confidence than the literature ultimately warrants. These challenges are especially significant for fraction division instruction specifically, since the measurement-based conceptual foundations that Wilkins and Norton argue are most important are precisely the foundations most directly relevant to understanding what it means to divide by a fraction, suggesting that even my own shift toward conceptual instruction in my classroom may have emphasized part-whole concepts more heavily than the research now suggests is optimal for fraction division understanding. I believe these counterpoints strengthen the review by introducing necessary complexity, but they also mean that my recommendations rest on a research foundation that is still evolving rather than settled.

Finally, while the personal reflection I include draws on twenty years of classroom experience and fourteen years specifically in fifth grade, I want to be clear that my classroom observations, however meaningful to my own professional development, do not constitute systematic research. The improvements I have witnessed in my students following a shift toward conceptual instruction are real and significant to me, but they are not controlled data. They represent one teacher's experience in one school context, and I offer them as a practitioner's perspective rather than empirical evidence.

I present these limitations not to undermine the recommendations I have made, but because I believe intellectual honesty about what we do and do not yet know is itself a form of respect for the teachers, students, and administrators this work is meant to serve. The evidence supporting conceptual-first fraction instruction is substantial, but it is not complete, and the practitioners who engage with these recommendations deserve to know that.

Conclusion

The evidence synthesized across this literature review converges on a single, consequential insight: how students are introduced to fractions shapes not only their immediate performance but their entire mathematical trajectory. Fraction division stands as the clearest illustration of this principle: students who encounter the invert-and-multiply algorithm without first understanding what division by a fraction means are left with a procedure they can execute but cannot explain, apply flexibly, or connect to the ratios and proportional reasoning that depend on it in later grades. When instruction begins with conceptual foundations; giving students the opportunity to understand *why* before learning *how;* it provides the cognitive scaffolding necessary for flexible reasoning, knowledge transfer, and sustained success in the increasingly abstract mathematics of middle and high school.

Yet this review has also been an exercise in intellectual humility. The counterpoints examined in Chapter 2 reveal that whole number bias persists in ways that resist even well-designed instruction, that students follow varied developmental pathways that complicate uniform prescriptions, and that the specific conceptual foundations emphasized in instruction matter as much as the decision to teach conceptually in the first place. Part-whole understanding, long the cornerstone of elementary fraction curricula, may itself be incomplete if not accompanied by measurement-based approaches, particularly for fraction division, where measurement-based conceptual understanding of how many groups of one fractional quantity fit into another most directly prepares students for the operation they are being asked to perform.

What remains clear throughout all of this complexity is the indispensable role of the teacher. Curriculum design alone cannot bridge the gap between procedural fluency and genuine understanding. That work requires educators who possess deep mathematical knowledge, who understand how students developmentally construct fractional concepts, and who can design and sustain learning environments where sense-making, not answer getting, is the central goal. Strengthening teacher preparation is not a peripheral concern; it is the necessary condition upon which all other instructional improvements depend.

Ultimately, the persistent difficulties students experience with fractions across decades of schooling are not inevitable. They reflect instructional choices about what to emphasize, in what order, and with what depth, that can be made differently. In fraction division instruction specifically, those choices, whether to build conceptual understanding of what division by a fraction means before introducing the algorithm, whether to use visual models that make the operation visible, whether to explicitly address the whole number bias that tells students division should make quantities smaller, are choices that every upper elementary teacher makes, whether consciously or not. The frameworks and findings synthesized here affirm that making those choices more deliberately, with both confidence in the evidence and humility about what remains unresolved, is among the most consequential work that elementary mathematics education can undertake.

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