Nash Equilibria Mappings

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1. What is a game?

Definition 1.1. N-player Game: Games with n players have n different strategy sets. Each n-tuple of strategies (one for each player) is accompanied by a payoff function which takes into account the state variables, which are those things which govern the payoff, but cannot be controlled by the players in general.
We will call S_i the strategy set for player i,
$M = S_1 \times S_2 \cdots \times S_n$ is the set of strategy vectors in an n-person game
$\mathbb{P} = (P_1(s), \ldots, P_n(s))$ is the payoff function.
That is, when each player $i \in 1, \ldots, n$ chooses strategy s_i resulting in strategy vector $s = (s_1, \ldots, s_n)$,
player i obtains payoff $P_i(s)$.

2. Mixed strategies

Definition 2.1. Mixed Strategy: A mixed strategy is one which associates a probability distribution over a subset of pure strategies.

Example: If player 1 can choose from a, b, or c, then a mixed strategy would be choosing a $\frac{1}{2}$ of the time and b the other $\frac{1}{2}$ of the time.
NB: Every pure strategy is a mixed strategy.

3. Establishing Equilibria

3.1. **Nash Equilibria.** Assume we have a game with n players in which each player i has a finite pure strategy set S_i, then we can create an n-tuple of pure strategies $[s_1, \dots s_i, \dots s_n], \forall s_i \in S_i, \forall i \in [1, n]$. The following two definitions use this framework to establish what we mean by a Nash Equilibrium.

Definition 3.1. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the $n-1$ strategies of the other players in the countered n-tuple.[N1]

Definition 3.2. Nash Equilibrium: We call a self countering n-tuple point a Nash Equilibrium. [N1]

4. An example…

	Y1	Y2
X1	3,3	0,1
X2	1,0	3,3

And now for something…

completely different.

5. Let's talk mappings:

Theorem 1. Let $M = S_1 \times \cdots \times S_n$ and $M^* = S_1^* \times \cdots \times S_n^*$ be two games associated with the payoff functions $\mathbb{P}$ and $\mathbb{P}^*$ respectively.
Define $F : M \to M^*$ by $F(s) = (f_1(s_1), \ldots, f_n(s_n))$
where $f_i : S_i \to S_i^*$ is a surjection $\forall i \in [1, n]$.
If $\mathbb{P}^*(F(s)) = A\mathbb{P}(s) + b$ where $A_{n \times n} \geq 0$ and $b_{n \times 1}$
Then $F(\mathbb{E}) \subseteq \mathbb{E}^*$ where $\mathbb{E}$ is the set of Nash Equilibria for M and $\mathbb{E}^*$ the set of Nash Equilibria for M^*.

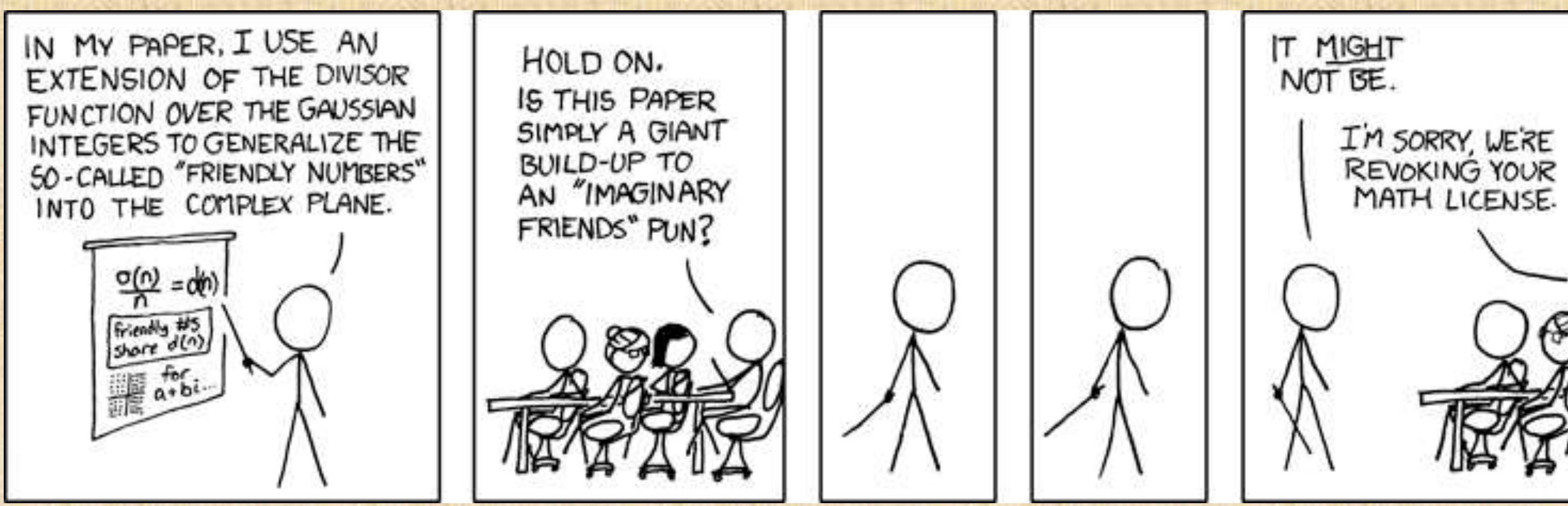
Proof in LaTeX,
So of course it's right…

- See board for sketch

6. Goals

Goals:

1. Find and classify games for which we can find a non-invertible matrix satisfying the above conditions.
2. Classify the games for which any A must have an inverse.



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- If we stipulate that the columns of A must sum to one, we are essentially redistributing the wealth of the payoffs among the players.
- The rank of the matrix directly corresponds to how many “groups” of exchanges there are.
- Future work will provide clarification of these groups and potentially extend the implications of the theorem.

Your Questions?

Thanks to:

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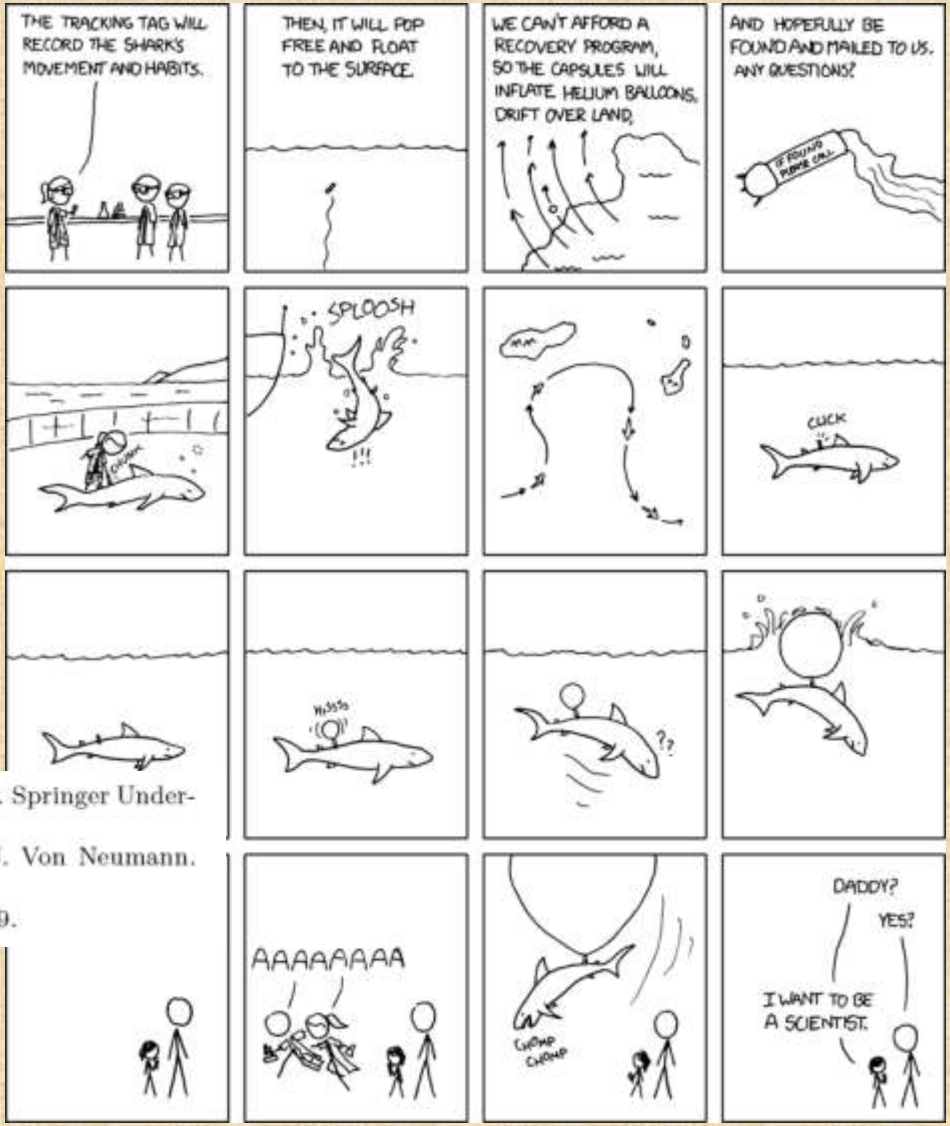
EPSCoR

XKCD

Me



References

[JW] Game theory: Decisions, Interaction, and Evolution. James N. Webb. Springer Undergraduate Mathematics Series 1995.

[vN] Solutions of Games by Differential Equations. G. W. Brown and J. Von Neumann. 1940.

[N1] Equilibrium Points in N-Person Games, John F Nash Jr. PNAS. 1949.

This presentation is not approved by persons above. Please note that this presentation may cause mental stress, especially in liberal arts majors. Here is a tiny picture of a frog in a hat. Thank you.